C. Recursively Defined Sequences

Sequences of the form $x_{n+1} = f(x_n)$ with a seed value for $x_1$ can be easily generated on the calculator. I will demonstrate using the example

\[ \left\{ \sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots \right\}, \text{ where } x_1 = \sqrt{2} \text{ and } x_{n+1} = \sqrt{2 \cdot x_n}. \]

1. Enter $\sqrt{2} \text{ ENTER}$
2. Enter $\sqrt{2 \cdot \text{ANS}}$ . The calculator returns the value $x_2$.
3. Pressing ENTER again re-executes the command in 2 using $x_2$ as ANS and returns $x_3$. Continue pressing ENTER to get other terms until the limiting value (2) becomes apparent.

I have seen this problem in maybe a dozen textbooks and every one of them uses the number 2; why not some other number? If you assign this particular problem, you could then ask students to guess the limit of the sequence with 2 replaced by 5 (or any other positive number except 2). They usually generalize from a single example and guess 5 which is incorrect. You can demonstrate the advantage of algebra over the calculator by finding a general expression for the limit with 2 replaced by an arbitrary positive number.

To graph the sequence above, you can use the following sequence of commands:

\[
\sqrt{2} \text{ STO S ENTER}
\text{For(N,1,20,1):PtOn(N,S):} \sqrt{2 \cdot S} \text{ STO S:End:DispG}
\]

For sequences defined by $x_{n+2} = f(x_n, x_{n+1})$ with seed values for $x_1$ and $x_2$, use the following:

\[ x_1 \text{ STO A:}x_2 \text{ STO B:}f(A,B) \text{ STO C:}B \text{ STO A:C STO B ENTER} \]

The calculator returns $x_3$, replaces $x_1$ by $x_2$ and $x_2$ by $x_3$. Continuing to press ENTER generates the sequence values. This is a bit complicated and will require some explanation, but I can't think of an easier way to do it.

For example, for the Fibonacci sequence, the commands would be:

\[ 1 \text{ STO A:1} \text{ STO B:A+B} \text{ STO C:B} \text{ STO A:C STO B} \]