15. **CURVE SKETCHING**

Graphing calculators and computers have changed the rules on teaching curve sketching. The developing of curve-sketching skills may no longer be an appropriate end in itself. Some reform texts do not even have sections on curve sketching. However, the relationship between the first and second derivatives of a function and the graph of the function is still one of the crucial concepts of calculus, and I believe that a few fundamental curve-sketching skills are desirable.

The relationship between the graphs of \( f \), and \( f' \), \( f'' \) can be nicely demonstrated by displaying the graphs (two at a time) on the same screen (see Section 11.E).

Other methods of reinforcing these concepts are:

1. Match graphs of functions with graphs of their derivatives.
2. Given the graph of \( f' (f) \), sketch the graph of \( f (f') \).
3. Give a graph of \( f' \) and ask questions about \( f \) (local extrema, intervals of increase/decrease, concavity, etc.).

The analytic approach is still very important. It is not too difficult to find problems for which a rough sketch of the graph is actually more easily obtained by analyzing the symbolic derivative than by using the calculator. Here are some possibilities:

\[
f(x) = \frac{1}{3}x^3 - \frac{99}{2}x^2 - 100x + 5
\]

Students will have trouble finding a good viewing window. Try \([-50, 180, 0]_x, [-200000, 500000, 0]_y\). It's easy analytically.

The next problem is courtesy of Rich Little.

\[
f(x) = 3x^5 - 25x^3 + 90x
\]

It's hard to get a graph which demonstrates all local max/min (try the settings \([-3, 3]_x, [-100, 100]_y\) and then zoom in on the flat parts using the BOX feature). It's fairly easy to do by hand -- the derivative is a quadratic in \( x \).

\[
f(x) = x^{4/3} + 4x^{1/3}
\]

This is easy analytically. There is a local minimum at \( x = -1 \), but unless students are careful about how they enter the function (the first term should be entered as \((x^4)^{(1/3)}\)), the calculator will not draw the graph for negative \( x \) (see Section 2.B.1).