VIRTUAL VERUS PHYSICAL: MANIPULATIVES IN THE MATHEMATICS CLASSROOM

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Introduction
Manipulatives have been a part of mathematics classrooms, especially in the early grades, for decades (Akkan, 2012; Johnson, Campet, Gaber, & Zuidema, 2012). Manipulatives were traditionally defined as objects which can be touched or moved (Akkan, 2012). These supplemental material were intended to help with student understanding and encourage students to think by themselves (Akkan). However, with the implementation of the NCTM Principles for School Mathematics (2000), a shift toward technology has emerged. In fact, these principles require teachers to use a variety of techniques to teach mathematical concepts which go beyond the abstract logic, algorithms, and theories most thought of when discussing these concepts. The technology principle recognizes that technology is an essential part in learning mathematics (NCTM).

Studies have shown that there are gains in student retention of material and understanding when manipulatives are used in the classroom (Carbonneau, Marley, & Selig, 2013). Even higher order thinking seems to increase (Carbonneau, et al.). An increase in content knowledge is seen in both students and teachers who use manipulatives and when a connection is made between the results of what is seen or modeled through the use of manipulatives and the explanation of the abstract properties of the problems (Puchner, Taylor, O’Donnell, & Fink, 2008).

As a result of the implementation of the NCTM standards, new initiatives have required classroom teachers to use a combination of virtual and physical or concrete manipulatives. Virtual manipulatives allow teachers to combine pictorial, verbal, and symbolic representations of problems more easily while still allowing students to move objects similar to physical manipulatives (Johnson, et al., 2012). Virtual manipulatives offer teachers flexibility and the ability to ask for explanations of reasoning since answers and feedback are more immediate than when physical manipulatives are used (Johnson, et al.). However, according to Akkan (2012), only a small percentage of pre-service teachers have information on virtual manipulatives and even fewer can use these confidentially (33% and 14% respectively). It is evident that there are some weaknesses to virtual manipulatives: less flexibility, less teacher input, harder for teachers to learn and use, more likely to encourage student memorization (Akkan, 2012; Johnson, et al.,
Effectively using manipulatives, both physical and virtual, greatly depends on the knowledge and ability of the teacher using these supplements. Teachers must have more than one session of professional development and must be able to identify manipulatives which provide deep correlation between the objects and the content being taught, offer a strong level of guidance on using the manipulatives, and identify the development level of students (Carbonneau, et al., 2013; Puchner, et al., 2008). Manipulatives seem most useful when students have experience using them, and time is invested into explanations on how to use them (Puchner, et al.). Once students know how to find the answer or reach a developmental stage where they are able to perform more formal operations, it seems that manipulatives are less useful and time is wasted on learning how to use the manipulatives (Carbonneau, et al.; Puchner, et al.). However, confidence in using virtual manipulatives and usefulness of these for enhancing learning of more challenging skills seems to increase as grade levels increase (Akkan, 2012).

Today, classroom teachers are challenged with using technology to teach the Common Core State Standards (CCSS). This charge is especially true in mathematics. Teachers are being asked to incorporate technology, project-based (PBL), and challenge-based learning (CBL) while teaching required standards to meet CCSS and state courses of study. For many years, professors of pre-service teachers have used hands-on manipulatives to enrich the understanding of many mathematical concepts in upper elementary and middle school. Educating pre-service teachers on methods to incorporate technology, PBL, and CBL in classrooms extends this challenge into the higher education classroom.

Physical and Virtual Manipulatives for Pre-Service Teachers
While many school districts are initiating a one-to-one initiative within their classrooms, some teachers are still searching for ways to teach the required objectives with non-existent funds. The current study required pre-service teachers to identify a common core standard from grades 6 through 8, develop or create a physical manipulative which could be used to enhance the instruction of the standard, identify a virtual manipulative which corresponds to the physical and is free, and create a worksheet using a free, online source. The students could use an online source or a free application compatible with smart phones or tablets.

As a part of the study, 33 students investigated fractions and decimals while 37 students were assigned to work within geometry. These students are all enrolled in the mathematics courses for pre-service elementary teachers but many of the standards overlap with those in middle school. The institution does not currently offer a methods course specific to middle school mathematics. A few students also worked on algebraic concepts such as expressions, equations, and inequalities. These students are enrolled in a mathematics methods course for pre-service secondary education majors. Some of these students also contributed to the online sources and applications for the fractions and decimals as well as geometry standards. However, they were not counted in the responses to the questions asked. The students were asked to reflect on their project by answering 8
questions:

1. Which do you prefer?
2. Which provided a better understanding of your topic?
3. Which gave you a clearer connection between the manipulative and the topic?
4. Which is more assessable to you?
5. Which is easier to use?
6. Which is more fun to use?
7. Which might you use again?
8. Which are you more likely to use in the classroom?

The results are given in Table 1 and Table 2. Students were also asked to provide the strengths and weaknesses of both the physical and virtual manipulatives. No students were in both classes which participated so the opinions are from a total of 70 students. The students who completed the fractions and decimals standards have not previously taken a course of this nature. However, those who completed the geometry project had already taken the course which addressed fractions and decimals. Since the responses were similar, it is useful to include a summary in Table 3 which reflects the results in terms of all 70 students.

Table 1

<table>
<thead>
<tr>
<th>Question</th>
<th>Physical</th>
<th>Virtual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Which do you prefer?</td>
<td>76</td>
<td>24</td>
</tr>
<tr>
<td>Which provided a better understanding of your topic?</td>
<td>67</td>
<td>33</td>
</tr>
<tr>
<td>Which gave you a clearer connection between the manipulative and the topic?</td>
<td>76</td>
<td>24</td>
</tr>
<tr>
<td>Which is more accessible to you?</td>
<td>42</td>
<td>58</td>
</tr>
<tr>
<td>Which is easier to use?</td>
<td>55</td>
<td>45</td>
</tr>
<tr>
<td>Which is more fun to use?</td>
<td>55</td>
<td>45</td>
</tr>
<tr>
<td>Which might you use again?</td>
<td>67</td>
<td>33</td>
</tr>
<tr>
<td>Which are you more likely to use in the classroom?</td>
<td>85</td>
<td>15</td>
</tr>
</tbody>
</table>

Note: Findings are given in percents and are rounded to equal 100%.
Geometry

<table>
<thead>
<tr>
<th>Question</th>
<th>Physical</th>
<th>Virtual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Which do you prefer?</td>
<td>78</td>
<td>22</td>
</tr>
<tr>
<td>Which provided a better understanding of your topic?</td>
<td>73</td>
<td>27</td>
</tr>
<tr>
<td>Which gave you a clearer connection between the manipulative and the</td>
<td>81</td>
<td>19</td>
</tr>
<tr>
<td>topic?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Which is more accessible to you?</td>
<td>43</td>
<td>57</td>
</tr>
<tr>
<td>Which is easier to use?</td>
<td>51</td>
<td>49</td>
</tr>
<tr>
<td>Which is more fun to use?</td>
<td>54</td>
<td>46</td>
</tr>
<tr>
<td>Which might you use again?</td>
<td>65</td>
<td>35</td>
</tr>
<tr>
<td>Which are you more likely to use in the classroom?</td>
<td>81</td>
<td>19</td>
</tr>
</tbody>
</table>

Note: Findings are given in percents and are rounded to equal 100%.

Table 3

<table>
<thead>
<tr>
<th>Question</th>
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<th>Virtual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Which do you prefer?</td>
<td>77</td>
<td>23</td>
</tr>
<tr>
<td>Which provided a better understanding of your topic?</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>Which gave you a clearer connection between the manipulative and the</td>
<td>79</td>
<td>21</td>
</tr>
<tr>
<td>topic?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Which is more accessible to you?</td>
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<td>Which is more fun to use?</td>
<td>54</td>
<td>46</td>
</tr>
<tr>
<td>Which might you use again?</td>
<td>66</td>
<td>34</td>
</tr>
<tr>
<td>Which are you more likely to use in the classroom?</td>
<td>83</td>
<td>17</td>
</tr>
</tbody>
</table>

Note: Findings are given in percents and are rounded to equal 100%.

All three tables indicate that students overwhelmingly prefer physical over virtual manipulatives. The only area where students chose virtual manipulatives was for their accessibility. In reference to preference, understanding, and providing a clearer connection to the topic, students thought the physical manipulatives were far better than the virtual manipulatives. Ease of use and providing fun connections or reinforcements were the areas where the two types of manipulatives were similar in liking.

Students were also asked to provide both strengths and weaknesses for the virtual and physical manipulatives they chose. Although the classes looked at different standards and mathematical strands, their comparisons were very similar. Students thought one of the strengths of virtual manipulatives was that they provided immediate feedback which has been shown to be useful for assessment. This opinion supports Johnson, et al. (2012) who found that virtual manipulatives were better than physical manipulatives in this respect. Students also thought virtual manipulatives provided easier access and a wider variety of choices including addition of sound which helps to hold interest. Johnson, et al. (2012)
also found this idea to be true.

Students agreed with Akkan (2012) that teachers lack understanding of using virtual manipulatives. This lack of knowledge can hinder the goal and outcome of the lesson (Puchner, et. al, 2008). Students also thought this deficit actually made using virtual manipulatives less effective and more time consuming.

Other student reflections concerning physical manipulatives included the following:

Strengths:
- Easy to manipulate and understand
- Force students to move around and be active
- Aid in “learning-by-doing”
- Easier to make a connection with attributes of size and length if the manipulative is hands-on versus displayed on a screen
- Enforce proficient use of tools of mathematics (ruler, protractor, compass, etc.)

Weaknesses:
- Take up valuable storage space, worn manipulatives have to be replaced, and easy to lose pieces; costly
- Limit number of pieces, hard to create, pricey
- Harder to control classroom behavior when using

Students shared the following feedback concerning virtual manipulatives:

Strengths:
- Less possibility of human error and less preparation time in respect to creating manipulatives
- Accessible for students to use at home
- Speed and ease of problem simulation

Weaknesses:
- Absence of using your hands; become lost in the repetition of just hitting keys on the keyboard
- Needed technology and hardware not always available
- Inaccessibility of on-line technology in classrooms
- Less interaction with others and less hands-on

Conclusion
It seems clear that many of the problems with using virtual manipulatives stem from lack of experience on the part of the teacher and pre-service teacher. Providing opportunities for pre-service teachers to use these supplements in similar projects such as this one will assist in building their confidence and encourage them to implement the aids in their own classrooms. In turn, this will provide a type of continued professional development for using manipulatives which Puchner, et al. (2008) recognize as a requirement for appropriate manipulative instruction. This type of project also creates opportunities for pre-service teachers to practice connecting the use of manipulatives with the standards they are required to teach. Helping them locate free resources will allow them to have a wider range of tools accessible for use with their students without having concerns about lack of available funding. The continuation of this project, as well as incorporating
suggestions by the higher education professors, will empower pre-service teachers with effective uses of both types of manipulatives.
Free Resources:

www.metric-conversions.org
www.coolmath4kids.com
www.gingerbooth.com
www.mathopenref.com
www.adaptedmind.com
www.funbrain.com
www.mathworksheetsgo.com
www.ixl.com
www.math-play.com
www.learner.org
www.iboard.com
www.math-play.com
www.math.uah.edu/stat/
http://nlym.usu.edu/
http://www.mathplayground.com/fractiongame/fractiongame.html
http://mrnussbaum.com/allgames/
http://www.factmonster.com/math/knowledgebox/
http://www.mathplayground.com/geoboard.html
http://www.mathgoodies.com/games/
www.funbrain.com
www.aaamath.com
www.mathsisfun.com

Geometry Pad by ZonMobile
Geoboard by The Math Learning Center
Protractor 1st by Apple
MathsJump
Math Contenders
Skitch
Three Doors
3D Coin Toss
Triangle Solve
Mystery Chronicles
Educreations
References


