Almost all books in College Algebra, Pre-Calc. and Calculus, do not give the student a specific outline on how to graph polynomials and rational functions. Instead, domains, intercepts, limits, continuity and asymptotes are detailed separately, and the student is left bewildered in a mathematical maze trying to find a way out. This paper uses all of the individual graphing ingredients and weaves them in a step by step procedure, where the student can go through it mechanically and without a hitch.

An interactive (bullet format) outline follows with two examples to demonstrate the procedure.

**Procedure:**
1. State the domain.
2. Find the Y-intercepts \((x=0)\), and the X-Intercepts \((y=0)\) the easy one in particular. **You can use the synthetic division to find the rational zeros for the given polynomial function. Basically, if \(f(c)=0\), then \((x-c)\) is a factor of \(f(x)\).**
3. For rational functions **ONLY**, find the asymptotes.
4. Perform the sign analysis.
5. Graph the function.

Now if we elaborate on step (3) for rational functions, we have: vertical asymptotes, horizontal asymptotes, and oblique/slant asymptotes.

**Asymptotes For Rational Functions**
1. **Vertical Asymptotes:**
   Whatever makes the denominator zero is your vertical asymptote, as long as you do not have \(0/0\). Remember that \(0/0\) means that you have a hole in the graph.

2. **Horizontal & Slant asymptotes:**
   Are the limits of the rational function as \(x \rightarrow \pm \infty\)
**Horizontal & Slant asymptotes**

Consider the following rational function:

\[ f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \cdots + b_1 x + b_0} \]

1. If the power of the numerator is the same as the power of the denominator (n=m), then the horizontal asymptote is \( y = \frac{a_n}{b_m} \).
2. If the power of the numerator is less than the power of the denominator (n<m), then the horizontal asymptote is \( y=0 \).
3. If the power of the numerator is greater than the power of the denominator by one degree (n=m+1), then the slant asymptote is \( y= \) the quotient of the division.  
   **Here the synthetic division can prove helpful when warranted.**

Notice that for rational functions, it should be very obvious that you cannot have horizontal and slant asymptotes at the same time.

**Using the Outlined Procedure Graph:**

\[ f(x) = (x - 1)(x + 2)(x - 3) \]

1. Domain: \( x \in (-\infty, \infty) \)
2. Y-Intercept: \( x=0 \rightarrow (0,6) \)
3. X-Intercepts: \( y=0 \rightarrow (1,0), (-2,0), (3,0) \)
4. Sign Analysis:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -\infty )</th>
<th>-2</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>
Using the Outlined Procedure Graph:

\[ f(x) = \frac{2(x^2 - 1)}{(x + 3)(x - 2)} \]

1. Domain: \( x \in (-\infty, -3) \cup (-3, 2) \cup (2, \infty) \)
2. Y-Intercept: \( x = 0 \rightarrow \left(0, \frac{1}{3}\right) \)
3. X-Intercepts: \( y = 0 \rightarrow (-1,0), (1,0) \)
4. Asymptotes:
   \( x \rightarrow \pm \infty, y \rightarrow 2; \quad y = 2 \text{ is a Horizontal Asymptote} \)
   \( x \rightarrow -3, y \rightarrow \pm \infty; \quad x = -3 \text{ is a Vertical Asymptote} \)
   \( x \rightarrow 2, y \rightarrow \pm \infty; \quad x = 2 \text{ is a Vertical Asymptote} \)
5. Sign Analysis:

\[
\begin{array}{cccccc}
-\infty & -3 & -1 & 1 & 2 \\
\infty
\end{array}
\]

| x     | - | - | + | - | + |
|-------|---|---|---|---|---|---|---|
| f(x)  | + | - | + | - | + |   |   |