

MULTIVARIABLE SPREADSHEET MODELING AND SCIENTIFIC THINKING VIA STACKING BRICKS

Scott A. Sinex
Department of Physical Sciences & Engineering
Prince George's Community College
Largo, MD 20774
ssinex@pgcc.edu

Abstract

Students measure the height of a variety of stacks of bricks and construct a mathematical model via a spreadsheet using linear regression. They predict the height of a 100 bricks. Then they go outside and measure brick heights in a real brick wall. Students quickly discover that mortar in between the brick is going to modify their model. They take measurements of bricks plus mortar and produce a second or revised model. Students compare models, consider random and systematic errors using a prebuilt spreadsheet simulation, and then develop a model to count the number of bricks in a specified area of a wall (and can verify). From multiple measurements of bricks and mortar joints in a brick wall, students can get a feel for real variation. In the end, a multivariable model is developed and explored as an interactive spreadsheet simulation. A version using Lego bricks and plates (mortar) is also considered.

Introduction

Mathematical modeling and simulation are important components of the *Common Core Standards in Mathematics* and the *Next Generation Science Standards* and are endorsed by AMATYC and the MAA. Plus they are tools to produce an engaging pedagogy and deeper learning (Honey and Hilton, 2011). This paper presents a simple model that allows for predictions followed by verification, and then revision of the model. The model is then expanded to consider error, both random and systematic, and then further expanded to include area determination and the computation of total bricks in a wall and its verification as well.

Here are the objectives of the activity and accompanying interactive spreadsheet:

- construct a mathematical model from the stacking of real bricks (or Lego bricks) and make a prediction - can be done from scratch to practice modeling or using pre-built “just-add-data” spreadsheets;
- verify the model by repeating measurements using a real brick wall (or Lego bricks with plates);

- discover the effects of mortar, difference in actual brick height, and measurement variation;
- revise the mathematical model to include brick height and mortar thickness;
- use and/or develop a simulation of both models - bricks only and bricks and mortar;
- investigate error, both random and systematic in a pre-built simulation, and;
- calculate number of bricks in an area of a wall using a pre-build simulation including a brick length model using collected data, and verify on a real brick wall.

This modeling activity is applicable to students from algebra I on up to college algebra. An understanding of goodness of fit using r-squared can be developed and knowing that linear regression produces the line of best fit to the data is all that is needed. Students could do the modeling from scratch in any spreadsheet program or use “just-add-data” versions in Google Sheets or MS Excel. The level of simulation building can vary from pre-built to having students or in-service teachers build them. In the end, students are exploring a multivariable model including experimental error and its simulation in the spreadsheet, a very real-world scientific process with a simple understandable system - bricks and mortar. Sinex (2013) describes a number of other simple linear modeling activities with simple manipulatives and then in Sinex (2015) introduces curvature and its detection with residuals.

Experimental Investigation

Using at least ten bricks, student groups can measure stacks of varying heights, measure to 0.1cm to collect data, record, graph, and derive a model by linear regression to get the line of best fit via a spreadsheet. They should also determine r-squared, a measure of the goodness of fit of the model to the data. The activity including the interactive Excel spreadsheet or Excelet is available at http://academic.pgcc.edu/~ssinex/excelets/Stacking_Bricks_Model.pdf. Readers may want to open the Excelet to view and explore the actual tabs or worksheets that are given in the figures that follow!

Once students have a regression line, they have mathematical model that they should write out in terms of the variables measured, not x and y. Hence the height, H, and the number of bricks in a stack, n, should yield: $H = \text{slope} \times n$, where the slope is the average height of the bricks used. Here is a typical result from a high school teacher workshop.

$$\text{Loose Bricks:} \quad H = 4.47n - 0.114 \quad r\text{-squared} = 0.9999$$

This is a case where the y-intercept should be zero but because of experimental error, a small value is typically obtained. We then use the model to predict the height of a stack of one hundred bricks.

Next data are collected outside at a real brick wall. Students discover quickly that mortar is going to cause their model to be incorrect. So they collect data again for a variety of brick stack heights and they take multiple measurements of brick heights and mortar thickness for different sites on the wall (See Table 1 for some teacher workshop data). Students will need to decide how to take consistent measurement with the stacks.

Table 1: Examining Real Variation in a Brick Wall

	brick height, cm	mortar thickness, cm
trial 1	5.8	1.1
trial 2	6.0	0.8
trial 3	5.7	1.5
trial 4	5.7	0.6
trial 5	5.8	1.3
mean	5.8	1.1
std. dev.	0.122	0.365
%CV	2.11	34.4

The multiple measurements clearly show that the mortar thickness has more variation than the brick height. Brick height is controlled in the US by ASTM standards while the mortar is placed by the bricklayer and may be checked on occasion. This teacher group picked a section of wall that had a very large variation. The mortar in other groups was always more variable than the bricks but not this large.

Here are results from multiple stacks on the wall for one group.

$$\text{Outside on wall: } H = 6.61n - 0.400 \quad r\text{-squared} = 0.9997$$

The new model shows a larger slope due to the brick height plus the mortar thickness. The model has more variation as judged by the larger y-intercept and slightly lower r-squared value.

Exploring with Spreadsheet Simulations

The spreadsheet is scaffolded to develop a number of concepts and reinforce an understanding of the mathematical model. After students have plotted their data for the loose bricks in the classroom and derived a linear model (equation of best fit), they can examine the Brick Model 1 Tab as shown in Figure 1 below. This simulation should get them to see that the slope of their model is the brick height. However, their model contains experimental error from the ruler measurements and any natural variation in the

brick height. Hence, the slope of their model is an average brick height for the loose bricks and this experimental error yields a small y-intercept and usually decreases the r-squared value by a small amount. Students should compare their results with classmates. Instructors may want to have students pool their data via Google Forms and Sheets as outlined in Sinex et al. (2016) as this makes a nice comparison.

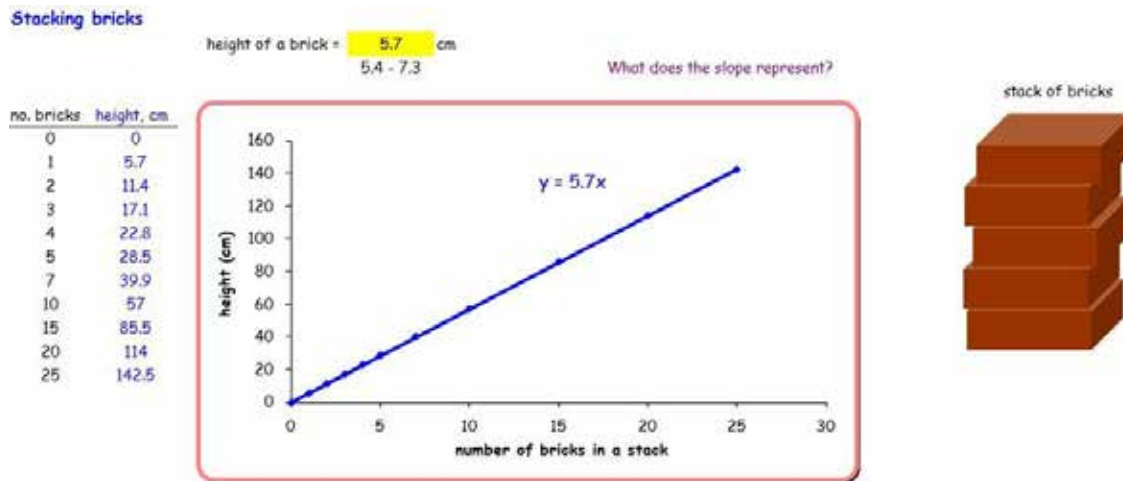


Figure 1: Bricks Model 1 Tab - loose bricks

The loose brick model proves to be too simple as students discover that mortar is used to hold the bricks in place. Figure 2 below illustrates the addition of mortar assuming one layer of mortar goes with each brick and this increases the slope of the model. Students quickly see that the slope is the sum of average brick height and average mortar thickness. Our loose bricks and the bricks in the wall used were also of different heights. This brings the multivariable aspects of the models in along with revising the model, a common scientific practice, to make it more realistic.

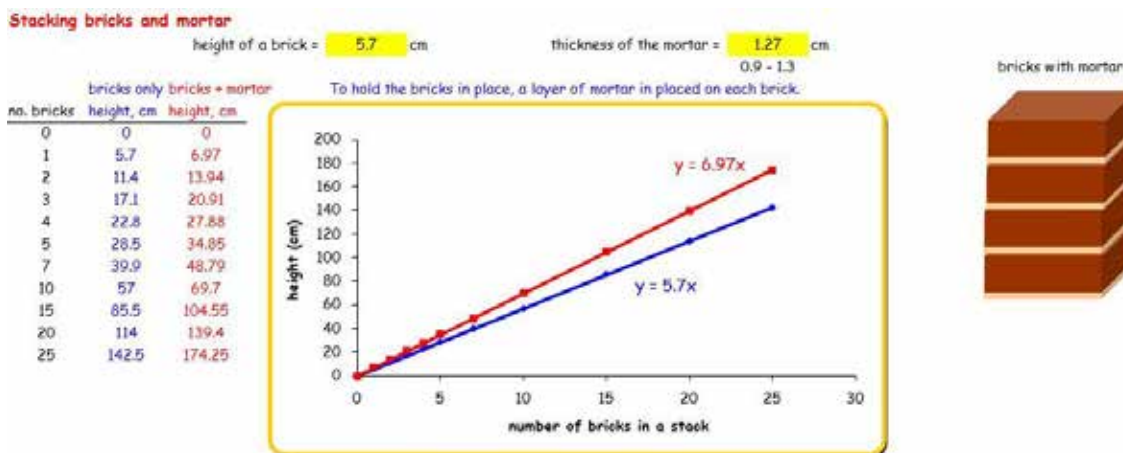


Figure 2: Bricks Model 2 Tab (red/upper line brick + mortar, blue/lower line bricks)

This simulation can easily be constructed in Excel or Google Sheets and may be an exercise for instructors to do with students. On the various tabs of the Excelets, if you click on the cells, the formulas appear and we have used named variables to help make the formulas clearer.

Investigating Errors

Determining and understanding measurement error is a critical part of any scientific investigation and is a mindset that must be developed in novice students. The Bricks Model Error Tab introduces random variation into the mortar thickness (scroll bar just above graph) and shows how it influences the model results (slope, y-intercept, and r-squared value). This is illustrated in Figure 3 and the random nature can be seen by hitting the F9 key a number of times to study the influence. The difference column is the absolute error and the percent error is the relative error. For more on the types of errors and their graphical influence on data and hence, the mathematical model, see Sinex (2005).

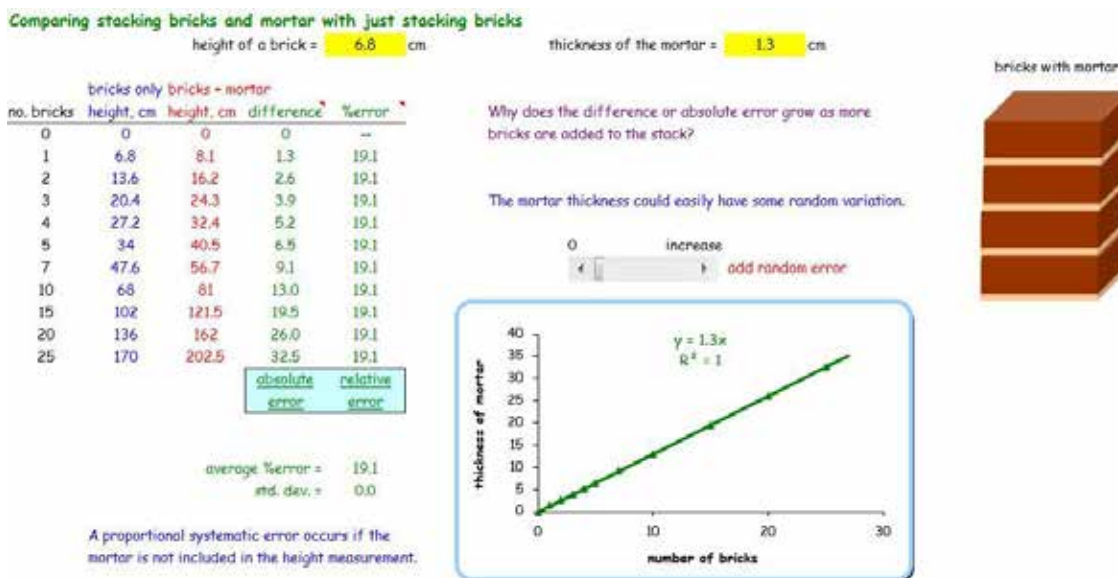


Figure 3: Bricks Model Errors Tab

On the Bricks Assessment Tab (not shown), a series of bricks of varying heights is plotted to see the scatter in the data and how it lowers the r-squared value. Hence the connection into variability of brick height. This variation is much larger than the actual variation in a batch of manufactured bricks. The point is to be sure that the average brick height as the slope of the model comes through to novice students.

A nice extension of the brick height model is to add in the brick length model. From the dimensions of a wall, and with the use of tracer points on the spreadsheet graph to interpolate, calculate the number of total bricks required as shown in Figure 4. Students can then go verify the calculation by counting on the wall. Verification is done by

measurement and not by calculation from a model. The classroom model gives a prediction by extrapolation!

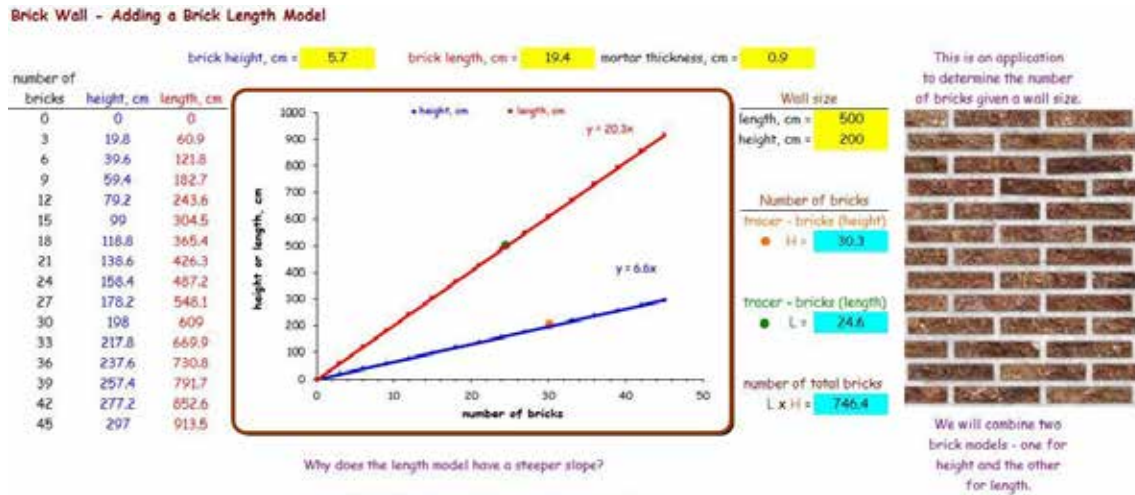


Figure 4: Bricks Model 3 (red/upper line brick length, blue/lower line brick height)

An alternative way to accomplish this activity is with Lego bricks and plates (serve as the mortar). A picture of a brick wall is required to discover the mortar. In Figure 5, the model is shown for Lego bricks and Lego bricks and plates (the mortar) plus ruler error and the possible error from what the top of the Lego brick is. These two constant systematic errors offer the possibility of cancelling each other out. How the origin (0, 0), which is not a measured datum point, influences the model can also be investigated.

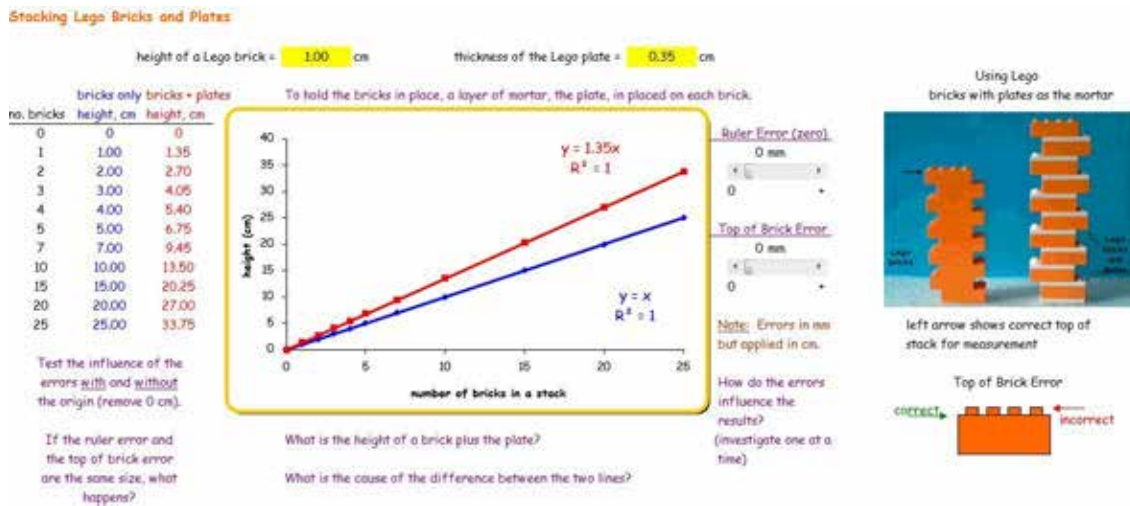


Figure 5: Legos Model Tab (red/upper line brick + plate, blue/lower line bricks)

Table 2 summarizes the results discussed above for the stacking bricks model and how it starts in a simple fashion by using loose bricks and builds to include bricks plus mortar and the addition of random and possible systematic errors into a multivariable model. An

extension to include the brick length and estimate of the number of bricks needed to build a brick wall is also covered.

The 2D wall area of bricks (discussed in Figure 4) can be plotted as a 3D plot in Excel as shown in Figure 6. The area plane is produced as a function of the three brick parameters (adjustable yellow cells) and the number of bricks in the horizontal and vertical directions. This illustrates the multivariable nature of this system.

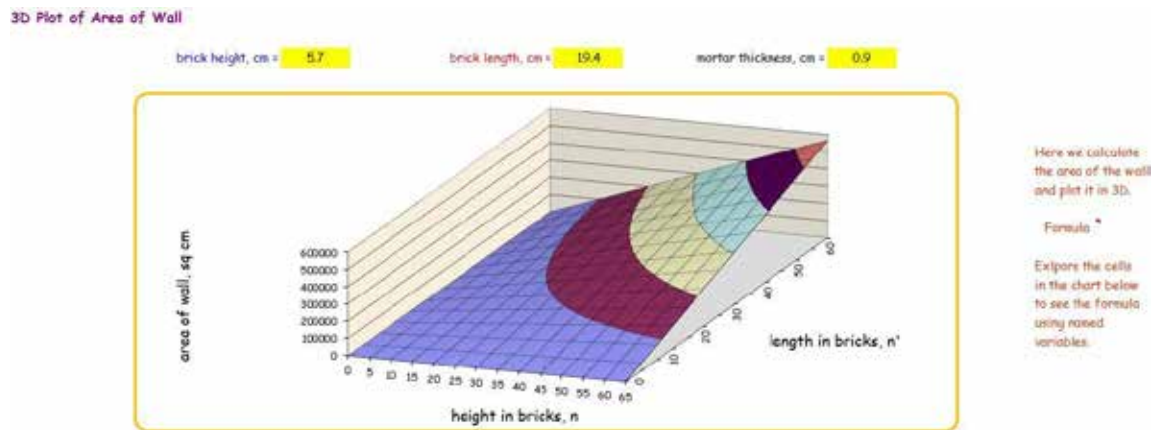


Figure 6: Wall Area 3D Plot Tab

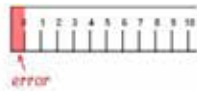

Teacher Workshop Feedback

Seventeen high school math teachers gave the brick height simulator a 4.2 out of 5 for overall use. Here are the responses for “What did you learn from this activity?” for four teachers that show the intended objectives.

- “It gave me an good example of modeling that students would understand and be able to complete.”
- “Great activity for my students for measurement and to understand why different groups get different data and results.”
- “How to get students thinking about many ways to have errors and how to take that into consideration.”
- “I learned how to create an excelet. I also learned the importance of being consistent with measurements. It is great to use with kids for learning about errors.”

All comments from the teachers were positive and the idea of using Lego bricks and plates was suggested by a teacher. One teacher did have reservations concerning using real bricks in class.

Table 2: Evolution of the Stacking Bricks Mathematical Model and Simulation

Type of Model	Equation	Variables	Goodness of Fit	Comments
linear model from stacking individual bricks	$y = mx + b_0$ in terms of measured variables: $H = bn$	$m = \text{slope}$ $b_0 = \text{y-intercept}$ $H = \text{height}$ $b = \text{brick thickness}$ $n = \text{number of bricks in stack}$	r-squared should be very close to 1 Is this model correct? Let's go & measure a wall	will have a small y-intercept due to measurement error & any variation in brick thickness
multivariable linear, revised model from actual measurements on brick wall - discover mortar	revised model: $H = (b + t)n$ slope depends on two variables	$H = \text{height}$ $b = \text{brick thickness}$ $t = \text{mortar thickness}$ $n = \text{number of bricks in stack}$	r-squared should be very close to 1	will have a small y-intercept due to measurement error & any variation in brick & mortar thicknesses
random error added as simulation of variation in variables	$H = (b + t + e_r)n$	adds random error, e_r (variation of brick &/or mortar thickness)	adds scatter to data points on graph; lowers r-squared value	random change in both slope and y-intercept (explore by multiple trials)
systematic error added - ruler error (simulated)  Lego - top of brick error (simulated) 	$H = (b + t)n + e_s$ Consider two cases: - without origin plotted (not a measured datum point) - with origin plotted (outlier)	ruler error, e_s , placement of zero mark not at end of ruler (very common) $e_s < 0$ not a problem using meter sticks ($e_s = 0$) also Lego brick top error ($e_s > 0$)	does not influence r-squared value if the origin datum point is NOT plotted if origin included, then lowers r-squared, slope, & y-intercept (rotates line)	causes a y-intercept as a negative value all measurements are low by this amount (causes a negative constant systematic error) Lego brick top - all measurements are high (causes a positive constant systematic error)
2D wall - area of bricks - two linear models to test number of bricks in wall of specified area (mark off area in centimeters with chalk)	$H = (b + t)n$ second linear model added: $L = (b_1 + t)n'$	new variables: $L = \text{length of bricks in a row}$ $b_1 = \text{brick length}$ $n' = \text{number of bricks in horizontal row}$ $n = n'$ or not	errorless perfect fits as simulation for measured models - r-squared should be very close to 1.	combining results give an estimate of the number of bricks in a wall and can verify models - use H & L to interpolate values of n & n'

Some Final Thoughts

Using simple loose bricks and a real brick wall (or Lego bricks and plates) and measurement, data can be collected, a model obtained, and then you can go verify the model and via the discovery of mortar, revise the model. This is real-world science process; it mimics the modeling cycle of the *Common Core Standards in Mathematics*, and deals with measurable variation that is understandable by novice students. An Excel simulation reinforces the model and allows experimental error, both random and systematic, to be introduced and provides an engaging pedagogy to support a data > model > simulation pathway for novice students by just using a spreadsheet. Depending on the group, instructors may build the simulations with students. For instructions on building interactive spreadsheets and other modeling activities, see the Developer's Guide to Excelets at <http://academic.pgcc.edu/~ssinex/excelets>.

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Classroom Resources

Discovery Channel Youtube Video - How to Make a Brick (for US manufacturing)

<https://www.youtube.com/watch?v=SbKvhHzn4hQ>

Specifications for and Classification of Brick - Brick Industry (describes ASTM Standards) <http://www.gobrick.com/portals/25/docs/technical%20notes/tn9a.pdf>

Number of Bricks in a Wall Calculator (black box approach)

<http://www.calctool.org/CALC/other/fun/bricks>