The United States Military Academy is one of the first Engineering schools in America. Founded in 1802 by Thomas Jefferson, it has been a pillar of education and research in the STEM fields for over two centuries. Throughout that time, West Point has been devoted to advancing science, technology, engineering and mathematical fields not only for academic pursuits but also to enhance the nation’s fighting force with a primary purpose of preparing the U.S. Army to win its wars.

This is probably one of the first things that people think of when they picture West Point – stoic, hard, immutable citizen-Soldiers trained to fight and win. And while some of those images may be accurate, it’s also not the full picture. One of the core tenets of the U.S. Army and thus West Point is selfless service. Soldiers understand that wearing the uniform is a service, an opportunity to put others before themselves. Cadets and faculty at USMA lead the way when it comes to service opportunities and strive to reach out to the surrounding areas, NYC and beyond to do give of themselves for someone else’s benefit.

With that in mind, a few of us in the Department of Mathematical Sciences wanted to take some time to explore the limits of our curriculum. Dr. Frank Wattenburg is a senior professor in the department and has developed some fantastic advances centered on using robotics in the classroom. We currently use Arduino robot kits for plebes (college freshmen to the uninitiated) and we hope to expand their use to further year groups.
These robotic kits cost around $300 and require assembly (batteries not included). After a few hours you’ll have a wheeled bot capable of accepting instructions i.e. scripts from a computer (Fig 1). In addition to a wheeled bot, you can add attachments that include a compass, a sonar sensor, an accelerometer and a color camera to name a few.

But what if we used these robots with younger students? What would we be able to do with them? What would some of the risks we would incur? What would the logistic plan look like? These were just some of the questions that came up from our initial discussions as we explored the idea of meeting with some Webelos, the senior ranks of Cub Scouts of America.

Let’s start with the students themselves; Webelos. Webelos are generally 4th and 5th grade boys. They have a lot of energy and can be distracted and/or lose interest quickly. This necessitates a plan to limit lecturing and a need to be creative. However you look at it, Soldiers can be the same way. We design our curriculum at USMA to be hands-on, leveraging technology to inspire the cadets. This felt like a win-win because our curriculum could easily be adapted for the target audience.

The next hurdle was one of scale – how many scouts would we have? A typical den (the smallest group) is between 4 and 12. With 2 instructors, 12 felt like it could be a little touch-and-go if we had any issues with the robots but at the same time it would be manageable, if a little chaotic. After a little debate, we decided that a den-sized element would meet our objectives (provide an interactive experience for the scouts with hands-on time while testing our curriculum in an unorthodox manner).

With 12 scouts as a maximum, we set up the room to accommodate 8 robots (Figure 2). We made the decision for this first outreach to limit the scope of the demonstration. We could have had the scouts build the robots from scratch, but that takes hours by itself with a lot of troubleshooting. Additionally, the robots rely on short scripts that also requires time to iterate and ensure the syntax is spot on. With these limiting factors in mind, we settled on a modified plan of having the scouts conduct a simple experiment using the robots and no coding.
Below are the lesson plans from which we modified the scout’s minicourse (figures 3-5).

Arduino Classroom Ready Materials - Pre-Calculus/Calculus I

Solving for the Speed of Sound (Modeling with Linear Equations)

Objectives:
1. Describe what is meant by mathematical modeling.
2. Know the definition of and be able to solve linear equations.
3. Know and be able to apply the three steps in the Mathematical Modeling Process: Transform, Solve, and Interpret.
4. Given an assumption used in the formulation of a mathematical model, explain why the assumption is necessary and reasonable.
5. Be able to use linear equations for solving application problems.

Assignment:
- READ: Section 1.1.1 - 1.1.4 in *Modeling in a Real and Complex World*
- READ: Section 1.5 in *Stewart Precalculus: Mathematics for Calculus*
- DO: *Stewart Precalculus: Mathematics for Calculus*: Section 2.1, Exercises 12, 14, 18, 113, 116, 119

Lesson Plan: 55 min Lesson

Materials Needed:
- Arduino Robots - Programs loaded and acoustic sensor removed (Enough for groups of 2-3 students)
- One tape measure, ruler, or meter stick per group (incremented to measure centimeters)

Activity 1: Introduction and Robot Construction
- Students start by attaching the acoustic sensor to the bread board.
- Students then follow instructions (printed/slide) to connect the four wires from the bread board (sensor) to the designated ports then attach the power supply cable.

Problem Scenario:
Acoustic location is the science of determining the distance to an object by analyzing the time it takes for a sound wave reflect off of the object and return to a sensor. In nature, animals such as bats and dolphins employ a similar technique, called echolocation, to aid in navigating dark environments and finding food. Sonar used by submarines allows the crew to navigate the ocean depths without the use of a window while geologists use sound wave refraction to map rock formations deep underground.

Your team just installed an acoustic sensor to your robot. The sensor is programmed to emit a sound wave and report the elapsed time (in milliseconds) for the wave to reflect back to the sensor. Collect data using the robot and calculate the speed of sound. Compare your answer to the theoretical value for the speed of sound, 343m/s, and justify any differences. Use the USMA Math Modeling Process to develop a linear model that predict the distance to an object based on the time it takes for a sound wave to leave and return to the sensor.

Activity 2: USMA Mathematical Modeling Process
Students implement the modeling process outlined in Figure 1.
1. Transform: Students begin by transforming the problem statement to math. They identify the given and unknowns, define variables, identify assumptions and state why they are reasonable and necessary. Students then develop a model to solve the problem. Students brief their plan to the instructor.

**Fig 3: Lesson Plan Pg 1**
2. **Solve**: Students apply mathematical techniques to solve the problem.

3. **Interpret**: Students translate their solution back into the context of the problem to ensure it “makes sense.” They also reflect on the stated assumptions and their impact on the solution.

**Activity 3: Data Collection**
- Students place the robot perpendicular to a fixed vertical surface (ie. wall or door) with acoustic sensor facing the surface. They record the distance from the surface to the acoustic sensors.
- Students then turn on the robot (using switch) to emit a series of sound pulses. The time readings may “jump” based on subtleties in robot orientation to the surface, so students should check their robot alignment and wait for the reading to stabilize.
- Students record the output from the LCD screen (or select the most prevalent value) and repeat the experiment at seven more times at different known distances.

**Activity 4: Analysis**
- Students create a linear equation with a data point using the relationship $Rate = \frac{distance}{time}$. They then solve for the speed of sound. This is repeated for each of their readings and the average of the results is compared to the theoretical value $340 m/s$, which is the speed of sound at sea level at $20^\circ C$.
- Students plot the data using a spreadsheet or analysis program and build a continuous model to represent the position function for the robot’s movement, Figure 2. Students can differentiate the model to predict the robot’s speed at a teacher specified time.
- **Interdisciplinary Concept**: Teachers can discuss the impact temperature and density have on the speed which sound travels through a fluid. Reference the ideal gas law, $PV = nRT$, from students’ chemistry courses. What is the speed of sound in water? Does the salinity of the water (fresh vs. salt) have an impact on the ability for sound waves to travel through it?
Table 1: Experimental Data

<table>
<thead>
<tr>
<th>Distance (cm)</th>
<th>Time (μs)</th>
<th>Rate (m/s)</th>
</tr>
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<td>340.64</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>915.4</strong></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Plot of Time for Sound to Travel Known Distance

Pseudo Code for Exercise

1: procedure SpeedOfSoundExperiment
2: Initialize:
3: pulseTime ← time for pulse to leave sensor and return
4: delay ← time between sensor readings
5: Loop:
6: loop measure sound pulse repeat
7: pulseTime = getDulse()
8: print ("Time: ", pulseTime, "ms")
9: pause delay
10: end loop

Fig 5: Lesson Plan Pg 3
The first thing that anyone is going to want to do with the robots when they sit down is to touch them, so that’s how we started. Taking inventory of the pieces is an important part of any operation (as Wesley states in “The Princess Bride” as he rattles off the assets). We had eight scouts including siblings show up and we asked them each to sit behind a robot station. They then ensured they had all of the appropriate pieces.

Once the inventory was taken, the scouts learned a little bit about the robots. We spent some time talking about science fiction where they would have seen robots like this. We talked about the Robot Trilogy by Isaac Asimov and roles that robots have played in movies. It was important to plant these seeds in their imaginations because oftentimes science fiction becomes science reality.

We then spent a little bit of time discussing the practical uses of robots in today’s society. In particular, we explored some ways that robots would be better than humans. The Army employs robots as drones in various operations and we brainstormed how the military would use a robot with wheels without sensors. They were pretty creative, and eventually we moved the discussion to what components a robot would need to be a self-driving car (the robot has wheels right?).

Now that we have planted the idea that we could have a self-driving car, we were ready to talk about the experiment. We discussed how a car would be able to drive itself and we landed on an acoustic sensor to determine the distance a robot would be from another car (Figure 7).

But how do we know if the sensor is working properly? We need to have a way to ensure that the robot is a safe distance from the preceding vehicle. Much like taking inventory, an important facet of an experiment or job is to have some way to ensure the device is working properly.

This is where the mathematics and experiment come in. We told them that we were going to install an acoustic sensor to each robot with the provided components and then run a series of distance measurements. After recording the data, we’d do some more discussion/exploration to see whether we could predict how far a robot was from a stationary object. The scouts were pretty excited at this point.
The process of adding the components to the robot is pretty straightforward. We have four colored wires that need to go into specific places and the acoustic sensor. With only eight scouts and parental supervision, the wires were easy to install. The breadboard (the white waffle-looking thing) simply requires the exposed wires to seat into it. There are no fasteners or clips to lose or break. The acoustic sensor (looks like the robot eyes) functions in the same way as it is secured to the robot via the breadboard in the front.

The scouts sometimes would miss the correct slots but for the most part they were self-sufficient with the help of our graphic displayed on the screen via a projector and PowerPoint (Figure 8). Once the robots were assembled, the scouts would read a time in milliseconds on the robot’s LCD screen. This time was how long it took for the robot to receive a sound wave once it was sent. This was refreshed each second.

We had targets (binders in Figure 9) and distances on a sheet of paper set from 0” to 5” in increments of 1” from the target. The intent was for the scouts to set the robot at a fixed distance and record the output. They would record six times on a sheet of paper and after all the measurements were taken, we gathered at the board to see what each scout recorded at each distance. We graphed the times vs the distances and we were able to discuss some sampling variances in addition to being able to start predicting the next points. The scouts were able to see the linear pattern forming and this formed the building blocks of talking about the speed of sound as a constant while introducing slope-intercept form for a linear model.

Overall, we felt like it was a huge success. The scouts were able to participate with robots while accomplishing a significant portion of their Supernova badge (a STEM-related award). From the scout parents, they were really impressed how attentive the scouts themselves were given that it was a day after school. Each scout was engaged and enjoyed their time here. More importantly we established a small seed of technological desire in the younger generation. Seeing mathematics in action is a powerful thing and who knows, we may have some future cadets in these scouting ranks.