1. Introduction

Inquiry-Based Learning (IBL) in mathematics can be a powerful tool for promoting deep understanding, critical thinking, proof writing, conjecture making, and many other aspects of what it means to “act like a mathematician.” In order to augment some of these desirable outcomes in my IBL classroom, I have embraced the use of SmartPen technology to accomplish many tasks related to these goals. In this report, I’ll show several ways in which the pens can be used in the writing, presenting, and critiquing of proof-based mathematics. We will also look at advantages, frustrations, and plans for the future.

2. What is Inquiry-Based Learning in Mathematics?

Inquiry-Based Learning (IBL) is a student centered method of teaching mathematics. There are many different types of IBL classrooms. All types of IBL classrooms have some characteristics in common:

- the students are primarily responsible for developing the material through some sort of guided experience (notes, a progression of theorems, etc.), rather than being told the material by the instructor;
- the students are given an opportunity to critique the work of other students;
- the instructor acts more as a coach for the students rather than a “fountain of knowledge.”

One of the most common IBL methods also goes by the name Moore method – or sometimes Modified Moore Method – which refers to Texas mathematician R. L. Moore who developed a distinctive style of presentation IBL. In these types of classes, the bulk of the class time is spent with students presenting proofs of theorems that they have worked on since the last class. The rest of the class then is charged with asking questions and constructively critiquing the proof presented in the front of the classroom. Classrooms vary as to whether students are allowed to work with each other outside of class, whether students are allowed to use resources such as textbooks when developing their proofs, or even the manner of discussion that occurs during the class (e.g., whether the class is allowed to make suggestions to further the proof or just to ask questions of the presenter). In any one of these situations, the students benefit not only from working through the proofs of theorems, but from seeing the solutions that other students give to the problems and often witnessing incorrect solutions or mistakes being made. It is from
these mistakes that meaningful classroom discussion can be generated and real learning occurs. Besides presentation IBL, other types of IBL structures exist, such as worksheet-based or POGIL (Process Oriented Guided Inquiry Learning).

In this report, I will describe a Real Analysis class I am currently teaching (Spring 2015 semester). This course is conducted according to a presentation based IBL (or Modified Moore Method) approach. That is, most of our class time is spent in discussions which are generated by student presentations. Almost never do I enter the classroom with prepared mathematical proofs to show the students. My voice tends to enter into the course via the problems that I ask the students to do. I have written all of the problems for the course and only release a few of them at a time, which allows the class to go in directions I had not anticipated at the beginning of the course. If a student comes up with an interesting conjecture or if a particular proof is not quite complete but rather needs a lemma in order to reach completion, then these items tend to show up on the students’ next batch of homework.

There are many documented benefits for using IBL techniques in the mathematics classroom. I will not make an attempt to catalog them here, but rather make reference to a paper containing many results. For example, Sandra Laursen [1] conducted a large-scale research project which showed significant gains when using an IBL classroom versus a traditional lecture classroom. These gains also reduce or eliminate the gender gap in upper-level mathematics. The students also tend to retain the material for longer.

3. Description of Project

As mentioned above, I am teaching a course in real analysis. This is actually the first semester of a two semester sequence in real analysis for which I will have the same students. There are nine students in the class. St. John Fisher College is a smaller college with a liberal arts core curriculum. The class meets three times each week (Mondays, Wednesdays, and Fridays) for 55 minutes.

Through some generous funding from the St. John Fisher College Department of Mathematical and Computing Sciences and a grant from the Educational Advancement Foundation I was able to purchase 10 LiveScribe Sky Wi-Fi SmartPens. Nine of these pens were issued to the students in the class and I have one for me. The pens require special paper, so notebooks and extra ink were also purchased for the students. The pens work by having a tiny camera near the tip of the pen which observes the dot pattern on the special paper and then uploads what is written on the paper to an online program called Evernote. LiveScribe makes other models of pens that work in different manners, such as using a cord or Bluetooth rather than Wi-Fi to transmit the information, and also using electronic repositories other than Evernote.

Before the beginning of the semester a plan was developed to effectively use this technology in the classroom and assess its usefulness. One of the aspects of this plan is that the students should do all of their work for the course using their SmartPen in their
special notebook. This way all of their work is uploaded to Evernote, where I can see it. Evernote creates folders (called notebooks) which can be shared with others, so all of the students’ online notebooks are shared with me. Also as part of this plan, a teaching assistant was hired to give feedback on the students work between classes. So when the class meets on Monday, the TA will come in on Tuesday afternoon in order to give a little bit of feedback to the students about the direction of their proofs so far. That feedback might be, “You're looking in the right place,” or it might be, “Remember to use the definition of that particular concept in your proof.” Furthermore, all of their homework is submitted using the SmartPen and graded and returned using Evernote. Because their Evernote notebooks are shared with me, I can annotate a copy of what they have submitted electronically and then place that copy in their online notebook. (This is the same manner in which feedback is given.)

Before moving on, I should mention that I am not the first one to use a SmartPen in this manner. Because of the Upstate New York Inquiry-Based Learning Consortium, I was able to acquire ideas from many IBL practitioners from my area. In fact, several IBL teachers in New York state are using the “between class feedback” approach, though not always with SmartPen technology. This project is basically an embellishment of the work of Patrick Rault at the State University of New York at Geneseo.

4. Shortcomings

Going into the semester, there was a plan for how these pieces of technology would be used. Not everything has gone according to plan, and even some of the things that have gone according to plan aren’t always optimal. Allow me to address some of the shortcomings of this project.

1. **Connection troubles.** The students, particularly at the beginning of the semester, claimed to have problems connecting to Wi-Fi in order to get their pens to upload the materials. They also complain that they don’t want to have to do their homework in Wi-Fi connected areas. I have had students use SmartPens in other contexts before and have never received so many complaints. After harshly indicating this during the fourth week of class, these problems nearly immediately went away! However, the first three weeks of class were somewhat rough. This lead to an initial inconsistency of expectation regarding the technology, and I’m afraid that I haven’t yet been able to recover from that.

2. **Pens take a long time to charge.** The pens come with a USB charging cable, but there is no adapter for a standard outlet. Therefore, in order to charge the pens one must use a computer or other USB device. Because the pens have an audio capability (which we really don't use this course) their batteries are actually quite large. However, the downside is that they take a long time to charge. It can sometimes take six or eight hours to charge a pen which is nearly out of battery. This usually isn't an issue, but it means that if a student’s pen is out of batteries, they cannot simply plug it in for a few minutes and then use it again or bring it to class – more planning is needed on the part of the student.
3. **Students hesitate to write in ink.** The students have been brought up in a culture of erasing their work when they would like to abandon it, and they were initially resistant to doing all of their work in ink.

4. **Students are hesitant to do all of the work in the same notebook.** Perhaps more understandable than the hesitance to write in ink is the hesitance to put all of their notes, scratch work, and assignments in the same notebook. Many of the students feel this is somewhat disorganized, as it is difficult for them to locate exactly where their homework is in the notebook (it is scattered between their notes and their scratch work).

5. **Not every upload is perfect.** Sometimes there are synchronization problems and not everything the student writes is upload perfectly. Fortunately, the smart pens act like regular pens (with ink!) so there is always a paper backup of everything that is done.

6. **The between class feedback is not happening as envisioned.** This is the most disappointing “reality” of the course so far. The students do not always contribute anything by the time the between class feedback is issued. When they do, and feedback is provided, the students receiving the feedback do not always take that feedback into consideration before the next class. I conjecture several reasons for this. One is the very real possibility that asking students to make headway on a problem between, say, the end of Monday's class at 2:30 PM and the time when feedback is given on Tuesday at 2:00 PM and then make more progress, given the feedback, between Tuesday afternoon and Wednesday afternoon’s class is just simply unreasonable for many of the students. Many of the students are perfectly willing to budget time to do their assignments, but cannot budget time every single day of the week to dedicate to this class. A second reason for this may be the fact that the TA has not been as reliable as I had hoped in terms of his availability. Finally, I suspect the biggest reason is that I have not given the students proper grade motivation to submit items for feedback and respond to the feedback that has been offered. It's too late in the semester now to go back and revisit grading policies, but should I try something different in the future I'll have to be careful to make sure that there is enough motivation for the students to engage with the feedback as intended.

7. **Evernote.** Many people use Evernote for various aspects of their lives. When used for the purposes of this project, it seems to be very lacking in many desirable features. For instance, the uploaded pen markings are stored as images in Evernote, so there is no search capability for the pages there. Furthermore, downloading the content from Evernote is difficult (there is no “export to pdf”); Evernote very much wants you to keep the files in specific file types so you need Evernote in order to view them. Thus, printing from Evernote is also awkward. Also, renaming notebook pages involves too many clicks and is not intuitive. Once renamed, sorting notebook pages can only be done according to a few built-in defaults (alphabetical, chronological), not sort-any-way-I-want. There are ways around this (by adding tags to pages, by adding subfolders, etc.) but these all involve asking the students to do more work simply for the sake of technology, rather than for the sake of actual mathematical learning. There are also many
problems on my end, such as the fact that the annotation feature only works on the downloaded program version of Evernote, not the mobile app or the web version. I have also experienced synchronization problems: I give students comments but then need to close the program abruptly and find that my comments did not properly synchronize with their folder.

5. Benefits

There have also been many benefits to using the Smart pens in my ideal classroom. These benefits extend beyond the original stated project plan.

Presentations. Because all of the student work is done using the SmartPen (in theory), a student can give a presentation by showing their Evernote notebook on the computer projector using Evernote's web interface. Furthermore, I can go into class knowing who is ready to present which problems, what types of errors I’m likely to see in the presentation, and think of what questions should be generated (or questions I might anticipate hearing). In this way, I am constantly going into class feeling a little more confident that I won’t get totally surprised by that the students do or do not know. IBL instruction gives rise to a lot of think-on-you-feet teaching, but this technology helps me give some forethought to questions before they get asked.

Multiple Platforms. Evernote is available for download, via a web interface (except on Android devices), or as a mobile app (including both Apple and Android). Therefore, the students have access to their work anywhere, even if they don't happen to have their pen and notebook on them. Students can receive notification when I put something in their notebook (usually feedback or graded homework) and react to it immediately.

Class Notebook. One thing that has gone over very well so far is the “class notebook.” This is an Evernote space that I have shared with the entire class where I can deposit exemplary solutions to various problems and an electronic copy of anything that was presented in class on a given day. This way students don't actually need to write down anything that is presented in class (though they do anyway). In theory, the students could also share ideas with each other through this notebook.

Digital Archive. Another benefit is that I have, in Evernote, a digital archive of everything in the course. This has allowed me to compare from semester to semester (and will be even more helpful in the future) and gives me easy access to materials for programmatic assessment. In fact, I have put in effort to score class submissions against rubrics for program learning goals, and this process was made much more effortless using this technology.

Aside from assessment purposes, I can use this in many ways in the future. I can have students critique proofs that were written by other students (in different semesters), I can create standards for the course that are level across semesters. I can look at how students
approach solving problems and use that to inform what I put on exams, what homework questions I ask them to do, and even shape the nature of the course.

While this list of benefits doesn't seem to be as lengthy as the list of drawbacks, it is to be understood that the drawbacks are mostly operational or logistical, whereas the benefits are deeper. In order to eliminate some of the drawbacks and reap the benefits, in the future I'll need to change the design of the course to make the between class feedback more effective.

6. Reference