USING THE HISTORY OF MATHEMATICS TECHNOLOGY TO ENRICH THE CLASSROOM LEARNING EXPERIENCE

Sarah J. Greenwald and Jill E. Thomley
Department of Mathematical Sciences
Appalachian State University
121 Bodenheimer Drive
Boone, NC 28608
greenwaldsj@appstate.edu and thomleyje@appstate.edu

In 21st century classrooms, teachers and students use a wide variety of technology for instruction and learning, and many likely do not recall a time when calculators, laptop computers, SMART boards, digital databases, the Internet, and other now-ubiquitous digital tools were not available. The development of this technology has a rich and interesting history, with contributions by people from many time periods and cultures, and its use has certainly impacted society both in and out of the classroom. At the same time, the evolution of technology has been affected by society, with innovations often growing from real-world needs and concerns. Introducing the history of technology into college-level classrooms and making explicit connections between tools, problems, people, and history may enrich students' experiences and allow them to connect more fully to mathematics concepts and the relationships between them. As educators, we have successfully used these ideas in a variety of mathematics and statistics classes. In introductory-level classrooms, history can provide a concrete context for mathematics content and help non-majors or other students who have had mixed experiences in prior mathematics courses connect or identify with the material. In senior-level capstone courses for mathematics and statistics majors, the history adds depth and allows students to further explore connections among the mathematics concepts and content they have learned.

History

Calculators, computers, and software have been used in mathematics and mathematics education for much longer than many people might imagine, and many individuals have contributed to the development of such technology throughout the centuries. Some measuring inventions are no longer used because of advances in mathematics and technology, which has led to changes in educational emphases, but their unique histories can provide interesting connections to people, places, and concepts. Mechanical calculators like astrolabes date back to antiquity, and scholar Abd al-Rahman al-Sufi wrote a detailed work on applications of the astrolabe in the 10th century. The sextant largely replaced the astrolabe for navigation in the eighteenth century, and has itself been overtaken by global positioning system (GPS) devices. The Greek Antikythera is another ancient mechanism that was used in conjunction with tables for mathematical calculations in astronomy. Many cultures had devices that could be broadly classified as an abacus, including the Babylonians, Romans, Chinese and Mesoamericans. The soroban, a Japanese modification of the Chinese abacus dating to approximately the 17th century, is still used in some schools today, and the Chinese abacus retains its popularity with in educational and business settings. Another tool that was historically important is the slide rule. During the first half of the 17th century that Edmund Gunter marked logarithms on a ruler, William Oughtred placed two sliding logarithmic rulers next to each other, and scientists could multiply numbers via the portable circular slide rule. The development of calculators and computers made the slide rule largely obsolete.

The Millionaire calculator, designed by Otto Steiger and commercially produced in 1893, is credited as the first commercial calculator that could directly perform multiplication. A little over a century later, students and teachers commonly had access to a wide array of programmable calculators, which automated many common calculations (such as square roots) that required much more time and effort in the not-too-distant past. The significant processing power of many modern calculator models, as well as use of QWERT-style keyboards and the ability of the user to write programs or apps, has blurred the line between simple mechanical calculation and true computers. Indeed, the 20th century was a period of rapid growth in technology, which accelerated in the latter half of the century. Broad-reaching historical events such as World War II and later the Cold War spurred major advances in the development of digital computers and calculators, many of which were first designed for military applications. For example, a major revolution in calculator technology occurred in 1958, when Texas Instruments (TI) engineer Jack Kilby invented the integrated circuit, also known as a 'calculator on a chip'. The TI CalTech four-function calculator debuted in 1965, less than a decade later. The TI-30 scientific calculator, which was released in 1976, was another leap forward. Some unit conversion calculators became popular in the 1970s as a way to introduce the metric system in the United States, and are still used in applications like construction. Perhaps the most successful business calculator was the HP-12C, introduced in 1981 and still produced by Hewlett Packard as of 2012. In 1985, Casio introduced the first graphing calculator, the fx-7000G. These and other companies continue to develop calculators that push the limits of both calculation and visualization on a handheld scale, as well as the seamless connectivity of handheld calculators and other types of computers.

Electronic computers developed along much the same timeline as handheld calculators. Charles Babbage is widely credited with inventing the first true computer in the 19th century, and his student Ada Lovelace is often cited as having written the first computer program. Just after World War II, John Mauchly and J. Presper Eckert created Electrical Numerical Integrator and Calculator (ENIAC), considered to be the first general purpose electronic computer. Mathematician John Von Neumann made important modifications to ENIAC to facilitate mathematical calculations. The first connections of what would become today's modern internet appeared at the end of the 1960s and evolved using the Internet Protocol Suite (TCP/IP), originally proposed by Vinton Cerf and Robert Kahn in 1974. Apple and IBM desktop computers debuted in the late 1970s and early 1980s, spreading rapidly through education and business, evolving first into portable computers, and then into other types of electronic devices such as tablets and smartphones in the 21st century.

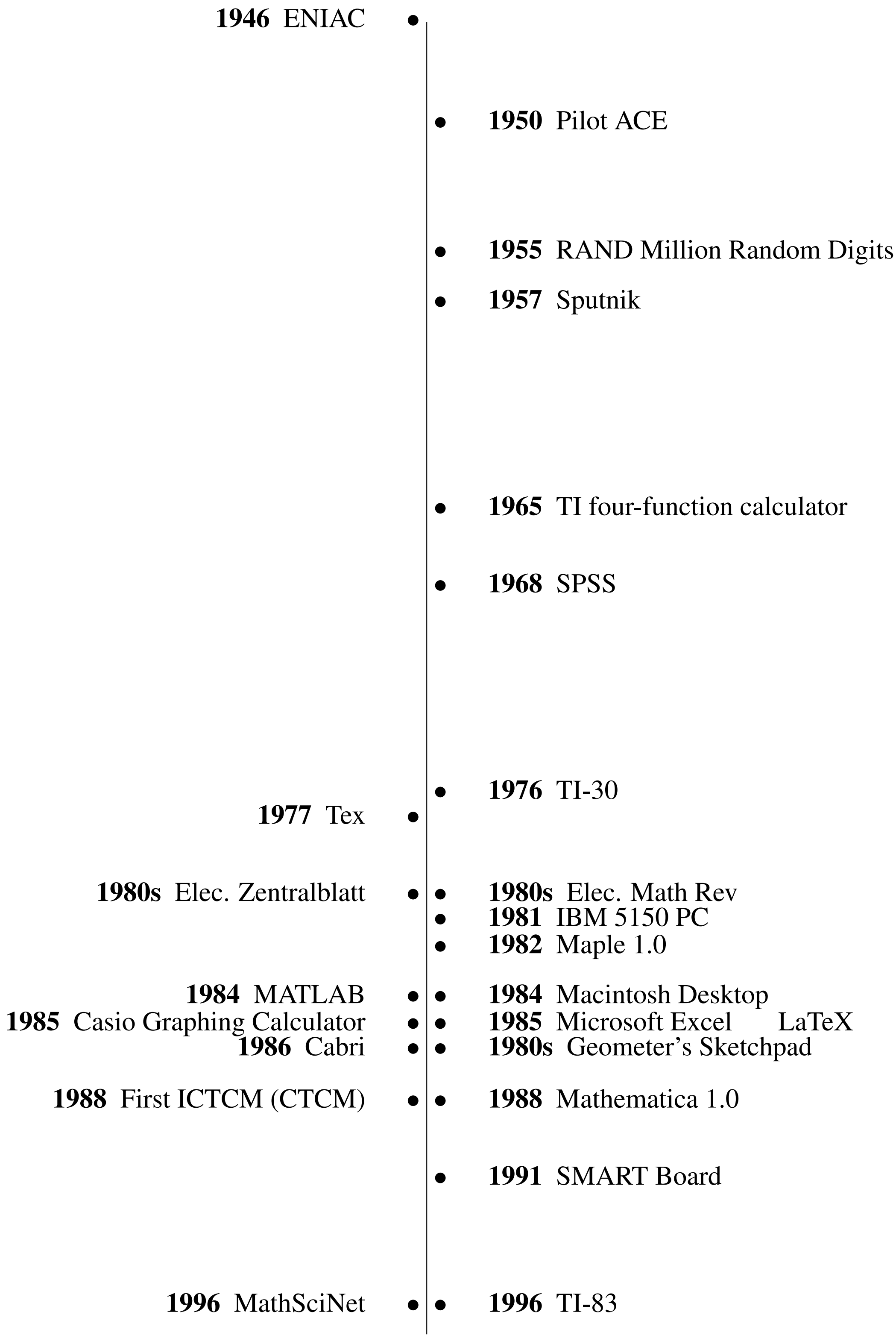

Figure 1: Select 20th Century Mathematical Software and Technology Dates

Not only are these devices used to teach and do mathematics, but mathematics is also required to create and evolve these devices, as well as the plethora of mathematical software available.

Mathematics software broadly includes computer, calculator, or web-based programs whose primary purpose is to graph, calculate, or manipulate mathematical data, either in numeric, symbolic, or geometrical form. These types of programs came into broad use at end of the twentieth century. The widely used Statistical Package for the Social Sciences (SPSS) was created in 1968 for mainframe computer systems, though it is now available for several platforms, along with many other programs such as R, an open-source package first released in the late 1990s. Popular computer algebra systems for mathematics at the college level include Maple, which was released in 1982, MATLAB, released in 1984, and Mathematica, released in 1988. Dynamic geometry software such as The Geometer's Sketchpad and Cabri also emerged in the 1980s. At the beginning of the 21st century, web-based Java and Flash applets have become more widely developed to teach mathematical concepts, and some complete commercial software suites such as StatCrunch are now completely web-based.

Overall, both individuals and organizations recognized the potential of calculating and computational technologies outside their original purposes or applications, and federal agencies began recommending investment in computers for research and educational use in the latter third of the 20th century. The 1980 publication *An Agenda for Action* by the National Council of Teacher of Mathematics recommended the use of computers and calculators in K-12 classrooms, notably before widespread introduction of desktop personal computers.

Classroom Examples

In 2004, The Mathematical Association of America's *Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide 2004* recommended that students should understand and respond to the impact of computer technology on course content and instructional techniques and that mathematics programs should lead people to learn mathematics in a way that helps them to better understand its place in society via its meaning, its history, and its uses. The American Statistical Association's 2010 *Guidelines for Assessment and Instruction in Statistics Education (GAISE) College Report* emphasized use of technology in classrooms not only for computational analysis of data, but also for developing conceptual understanding. Groups and individual instructors seek to implement these ideas in a variety of ways. We have observed that historical activities help show that mathematics is a living discipline and it easy to bring in history in a wide variety of classes to contextualize concepts and enhance student understanding, as in the examples below.

Linear Algebra

In linear algebra classes, the Hill Cipher is an application of matrix multiplication and inversion. Showcasing images of Louis Weisner and Lester Hill's mechanical *Message Protector* from 1929 can help students connect to the material. The patent papers and diagrams of the mechanical device with six gears can be found on the U.S. Patent and Trademark Office website by searching for U.S. patent number 1845947. Alasdair McAndrew has written a paper detailing the use of the Hill Cipher to teach cryptographic principles [9]. Some linear algebra texts include a section on coding using matrices. For instance, in Larsen and Edward's *Elementary Linear Algebra* an invertible matrix A is used as follows:

$$A[\text{original message}] = [\text{coded message}]$$

The matrix A^{-1} decodes a message. Students can code and decode messages by-hand for 2×2 examples and use a computer algebra system to code or decode messages using larger matrices. To complement this section, we show the students the patent diagrams. We briefly mention the idea of modular arithmetic (modulo 26 in this case, for the number of letters in the alphabet), and then we discuss the vulnerability of the method to those that intercept enough pairs because of its linearity. Our students report amazement at the existence of the methodology long before computer algebra software and they are very interested in the images of the device and the gears and chains. Some students choose to investigate linear methods of cracking the cipher in a final project.

Geometry

Many students in our geometry course are future teachers and member organizations recommend including historical perspectives for this student population (e.g. [3]). In addition, in the 20th century, geometry education was fundamentally transformed because of computers, calculators and other devices. Geometry for navigation, like spherical trigonometry computations, was built into computer programs or global positioning systems and so the related topics were eliminated from the curriculum. Two school programs that originated in the 1980s and remain in use at the beginning of the 21st century are Cabri Geometry and The Geometers Sketchpad. Jean-Marie Laborde headed a team to develop Cabri in order to explore geometric relationships. Nicholas Jackiw created Sketchpad as part of a visual geometry project headed by Eugene Klotz and Doris Schattschneider. Geometry educational software continues to be developed, including open source versions.

In class, we mention the technological history and introduce related quotations. One quotation is from Nicholas Jackiw: *I didn't want things that made* [the program] *seem like it was representational... They* [labels] *turned it into an illustration, whereas I wanted it to be a world* [15]. Another quotation we share is from Colette Laborde: *The idea of movement in geometry is not new - the Greek geometers devised various instruments to describe mechanically defined curves - but the use of movement was nonetheless prohibited in strict geometric reasoning for reasons that were more metaphysical than scientific* [15]. Using these quotations can help students think of dynamic geometry software programs as human endeavors and we especially like these quotations because we continue the themes throughout the course. We refer back to Jackiw's quotation and the idea of geometries as worlds in discussions related to axiomatic systems for Euclidean and non-Euclidean geometries. The theme of movement begins in our discussions of Euclid's proof of side-angle-side. We discuss what he might have meant by the (translated) term superposition and how he generally avoided this method. Transformations are a topic in the catalog description so we investigate movement as a fundamental part of the course. At the end of the semester we conclude this theme by mentioning Felix Klein's Erlangen program, in which a space is now understood through algebraic techniques by the geometric transformations that preserve it.

Statistics

The philosophy and practice of teaching introductory statistics has changed over the past two decades with the availability of advanced calculators and interactive computer software, which can quickly perform most needed calculations. Such technology has allowed instructors to shift their focus from computational methods, which were often tedious and occupied a great deal of classroom time, to the interpretation of data, which has been encouraged by professional groups [1]. However, some common methods in statistics, which have become standard across nearly a century of practice, evolved in part because of the limitations of early 20th century technology. Discussing some of this history can help students understand concepts, make connections between methods, and be flexible to evolutions in statistical methodology.

One example of the role that the history of technology plays in statistical practice is the statistical tables and the standard use of $\alpha = 0.05$ as the criterion for rejecting a null hypothesis. This can seem arbitrary to students, who do not understand that this 'magic number' arises from the original computations in the tables or what relation α has to p-values generated by technology. There also can be some disconnect at times between computer methods and the hand-calculations using tables that occupy appendices in many introductory textbooks. These tables can be confusing for students, who may not understand how they relate to the information provided by technology. Asking students to consider the role that technology has played in the past may help them. We find that most students are able place the origin of the electronic computer to the mid-20th century, but are unfamiliar with how calculations were done prior to that time (though many will guess slide rules). A quote by Karl Pearson, from the seminal 1914 book *Tables for Statisticians and Biometricians*, may be used to initiate discussion.

> I am very conscious of the delay which has intervened between the announcement of the publication of these Tables and their appearance.... First the great labour necessary and secondly the great expense involved in the calculation of the Tables... Logarithmic tables are relatively little used by the statistician to-day, which is the age of mechanical calculators [13].

What sort of mechanical calculators does Pearson mean? After speculation by students, we show an image of the Millionaire calculator, which debuted in 1893. They are typically amazed and often laugh out loud at the idea of a suitcase-sized calculator. Several questions can be used to connect past and present. For example, how could we re-create a textbook table using the Millionaire and how many people would we need? How would the process be different using modern computers? This requires thinking about exactly what the tables represent and the way in which information is produced by software. In this context, students also consider what kinds of choices must have been made (e.g. 0.05 versus some other critical value) with respect to the information made available to students and professionals, usually in the form of books. Similar conversations may be had regarding ideas like hypothesis testing (Why do we use approximate tests?) or pseudo-random number generation (Can a number generated by a computer be truly random?).

First Year Seminar

We teach a first year seminar on breakthroughs and controversies in science and mathematics. One of our texts for the course is Easton's *TAKING SIDES: Clashing Views in Science, Technology, and Society* and we let the students choose the topics they wish to discuss. During the Fall of 2009, students chose *Can Machines be Conscious?* The reading presented a 'yes' and a 'no' side. After a class discussion on the reading, we brought in some historical perspectives including a quotation from Alan Turing: *A computer would deserve to be called intelligent if it could deceive a human into believing that it was human*. Next we examined the 1950 Pilot ACE (Automatic Computing Engine), the oldest general purpose electronic computer in Britain, which adapted from a design by Turing, and we 'talked' to the computer A.L.I.C.E., who was noted as the first to have played Turing's Original Imitation Game (in 2005). Each year the students complete a historical timeline research project that highlights scientific and mathematical breakthroughs and controversies. The students present their research to each other in a session that is modeled after poster presentations at research conferences. Over the years, numerous students have chosen to explore numerous topics related to the mathematical history of technology, including robots, artificial intelligence, video games, space travel, the Internet, nanotechnology, and solar technology. For more information on the project, see http://cs.appstate.edu/∼sjg/class/fs/timeline.html. One of the course goals is to examine how personal, historical, and cultural perspectives affect the discovery and generation of knowledge, and we satisfy this goal through these kinds of activities.

Senior Capstone

Students in senior-level capstone courses are often expected to conduct research and communicate their findings in written form. Many of our capstone goals explicitly tie together history, communication, and technology:

- Select and use hardware, software applications, databases, and other technologies effectively for both inquiry and communication.
- Understanding responsibilities of community membership
- To relate mathematics (or statistics) to other disciplines and society

- To reflect on mathematical ideas from the past

We discuss the history of collaboration, research and communication in mathematics, from historical exchanges by letter to the kinds of real-time sharing now made possible by technology, and the evolving standards of professional communication and publication. This includes abstracts of research articles that once existed only in printed form. For example, reviews began appearing in journals like *Zentralblatt für Mathematik* and *Mathematical Reviews* in the first half of the 20th century. Since the 1980s, electronic versions of these reviews have allowed researchers to search for publications on a specific topic. In 2010, *MathSciNet*, the electronic version of *Mathematical Reviews*, listed more than 2 million items and more than 1 million links to original articles. In 2011, the database *Zentralblatt MATH* listed more than 3 million items from approximately 3500 journals and 1100 serials dating back to the 1800s. We ask them to consider what research was like before electronic sources and how much even that has changed within their lifetime. Also, in terms of creating professional documents, we discuss how TeX was created by Donald Knuth in order to typeset mathematical research: *I had spent 15 years writing those books, but if they were going to look awful I didn't want to write any more* [8]. We also show the students a Knuth reward check, which shows them one of the ways the scientific community had a role in the development. Student also use electronic databases to access seminal papers

Conclusion

While many students and teachers at all levels of mathematics education have embraced the use of technology and software in the classroom, this movement has not been without its 'growing pains' and controversy. One early example occurred in 1993, when students at the University of Pennsylvania complained about frustrations with Maple in calculus classes. They cited a lack of support and faculty expertise, and some students even wore shirts printed with the slogan "F- Maple". This attracted national attention. Even now, use and implementation of software in classes continues to generate debate among parents, students, and educators regarding the balance between students exploring concepts and solving problems using by-hand methods versus computers. For instance, the 2008 National Mathematics Advisory Panel cautioned:

> Despite the widespread use of mathematical manipulatives such as geoboards and dynamic software, evidence regarding their usefulness in helping children learn geometry is tenuous at best. Students must eventually transition from concrete (hands-on) or visual representations to internalized abstract representations. The crucial steps in making such transitions are not clearly understood at present and need to be a focus of learning and curriculum research [11].

There are also ongoing questions regarding how much teaching time should be focused on instructing students in the use of software versus addressing concepts, though some would argue that there is no clear divide between the two. Some people pose even larger questions about the essential relationship between mathematics and technology, such as the legitimacy of computationally-based proofs. When such concerns are considered in isolation from the history, it is easy to overlook that fact that use of technology in mathematics education is by no means a wholly modern innovation; it is simply that the tools we use have evolved over the centuries. Further, the evolution of technology used in mathematics classrooms has frequently been inspired by practical, real-world needs and concerns, or by mathematics researchers exploring new concepts and connections. From that perspective, technology, society, and education are inextricably intertwined and build upon one another. This leads us to the notion that students' understanding of mathematics and their personal connections with mathematics concepts, as well as their understanding of relationships between the various elements, can be enriched by discussing the history of technology in college-level classrooms and explicitly making connections between tools, problems, people, and history. Looking to the future, it seems clear that technology will continue to play new and innovative roles in mathematics education. For example, in the same way that software and technology has facilitated exploration of advanced concepts by lower-level students, open-source software increasingly gives students at various educational levels the opportunity to be involved in the creation of mathematics software and technology.

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