GEOBOARDS TO GEOGEBRA: MOVING FROM THE CONCRETE TO THE ABSTRACT IN GEOMETRY

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Abstract

The paper discusses about the importance of providing a transition from hands-on geoboards to developing abstract concepts using GeoGebra. GeoGebra is an emerging technology in Florida, the United States, and around the world. GeoGebra can assist in developing a deeper understanding to geometric concepts in the elementary (K-6) mathematics classrooms. Geometry activities are provided that can be used at the elementary level with the emerging software, GeoGebra, found at: geogebra.org.

Introduction

Focusing on geometry first in the mathematics curriculum is critical. Young students have a stronger background with geometry due to their surroundings and objects that they play with as a baby and child. Fostering these strengths is critical in developing important math concepts at the elementary level. Reys (2009) discusses the following aspects as to why we teach geometry:

1. Geometric knowledge, relationships, and insights are useful in everyday life and are connected to other mathematical and school subjects - it surrounds us.
2. Geometry helps us to represent and describe in an orderly manner the world in which we live.
3. Children are naturally interested in geometry and find it intriguing and motivating.
4. Children's spatial abilities frequently exceed their numerical skills and tapping these strengths fosters an interest in math and improves number understanding and skills later on when starting with geometry first.

When teaching geometry and mathematics, educators need to be reminded of the critical fourth aspect. We want excite students with math so focusing on students' strengths fosters stronger interest. Often it may be best to start teaching young children geometry first as numbers are considered to be more abstract and difficult to learn. By focusing on one of the most concrete branches of mathematics first, geometry can benefit students' whole view of mathematics and their attitudes towards learning it.

Concrete to Abstract

Piaget talks about this idea and the process for teaching in his distinguished model. Piaget's Four Stages of Cognitive Development (Figure 1) and discusses that young people work through these stages as they advance in their learning as follows:

1. Sensorimotor Stage: (Infancy - birth to age 2). Children experience the world through movement and senses.
2. Pre-operational Stage: (Pre-school - ages 2 to 7) Acquisition of motor skills but thinking is done in a non-logical.
3. Concrete Operational Stage: (Childhood - ages 7 to 12) Begin to think logically but are very concrete in their thinking. Intelligence is demonstrated through logical and systematic manipulation of symbols related to concrete objects.
4. Formal Operational Stage: (Adolescence - age 12 onwards) Development of abstract reasoning. Children develop abstract thought and can easily conserve and think logically in their mind. Intelligence is demonstrated through the logical use of symbols related to abstract concepts.

These four stages are widely used and accepted in the education field and for teaching mathematics around the world. Students need to always start with concrete objects first. Nishida (2007) in the book, *The Use Of Manipulatives To Support Children's Acquisition Of Abstract Math Concepts,* found based on empirical studies that math manipulatives did have a strong impact on student success in understanding mathematics. The National Council of Teachers of Mathematics (NCTM) has advocated the use of manipulatives for several decades now in various publications (1989, 2000, and 2006).

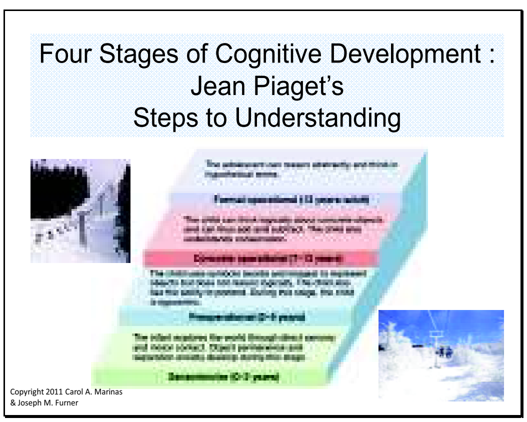

Figure 1: Four Stages of Cognitive Development

Concrete to Abstract Activities

Concrete	Abstract
Counting 3 marbles with 2 marbles	3 + 2
Measuring the perimeter of a circle with radius of 4 inches	C = πd C = π(8) or about 25 inches
Find the number of times of getting a 2 when rolling a die 90 times	P(getting a 2 on a die) * 90 = $\frac{1}{6}$ x 90 = 15 times
Creating geometric shapes using rubber bands and a geoboard	Use Virtual Geoboard Web Sites or GeoGebra

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Figure 2: Examples of Concrete and Abstract Math Ideas

The NCTM (1989, 2000, and 2006), Reys (2009), and many renowned researchers have recognized the importance of teaching using concrete items before going on to the abstract understanding of mathematical ideas. Some examples are found in Figure 2.

As math educators, it is critical to build bridges connecting understanding in mathematics. By using Piaget's ideas, we teach by using concrete and hands-on activities, then connecting to representations in drawings and diagrams, and finally connecting to the symbolism and abstractions at the abstract level. The Concrete Representational Abstract, or the CRA Model, is the most widely used method for teaching mathematics to students in schools today. The CRA Model follows the following three steps:

1. Start with the **C**oncrete using hands-on manipulatives
2. Move to **R**epresentational models in diagrams
3. Lastly, connect to the **A**bstract symbolism where student understand and function at an abstract level completely

Providing a concrete role in developing geometry understanding, a geoboard is a board covered in a grid of pegs that is used to construct geometric shapes and explore their relationships. (Figure 3) Even though there is technology like the vitual manipulatives, Scandrett (2008) in research concluded that the geoboards still need to play an important concrete tool in the math classroom. Hands-on experiences are seen as very important in the cognitive development of young people.

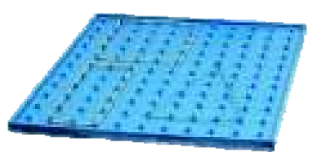

Figure 3: Geoboards

Moving from the concrete then to the semi-concrete or representational level involves using something similar to the geoboard, but a virtual version. Interactive experiences from websites like the National Library of Virtual Mathematics Manipulatives provide representational models of geometry. While not physically hands-on, the geoboard is shown online virtually allowing students to make the leap from concrete to semi-concrete or representational. Appendix B has online interactive web sites on this topic. Hwang, Su, Huang, & Dong (2009) found that the virtual manipulatives used with SMART boards enhanced students representational understanding of learning mathematics.

Students may move from using the geoboards, to the virtual geoboards, then finally to GeoGebra. When elementary teachers use GeoGebra with students, literature suggests that they turn off the Algebra View and leave only the Grid View on the screen. The GeoGebra Grid will focus the students on a virtual geoboard. Fahlberg-Stojanovska, & Stojanovski (2009) determined that using GeoGebra is a motivating for students and helps them learn at a higher level while exploring and conjecturing as they draw and measure. Rosen & Hoffman (2009) established the importance of integrating both concrete and virtual manipulatives into the primary-age math classroom. Furner & Marinas (2007) found that children easily transition to the abstract when using geometry sketching software when you first use geoboards and then software *Paint* before going directly to the sketching software. Figure 4 shows an example of the software, GeoGebra. Appendix A shows several examples young children can explore with GeoGebra. Appendix B provides online web sites on geoboards and GeoGebra.

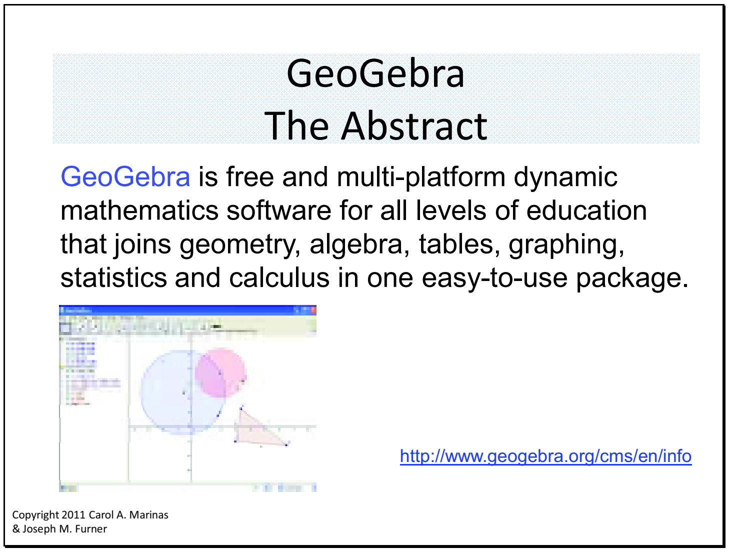

Figure 4: GeoGebra Example

In comparing geoboards to GeoGebra, students can see many similarities like the grids. Depending on comfort level and cognitive understanding, students will feel more comfortable moving from one stage to the other, the Figure 5 shows a comparison of the two:

Geoboards	**GeoGebra**
• Use as hands-on activity to create real world understanding • Understand measurements with a ruler • Time consuming • Discipline issues and training to use	• Uses abstracting thinking to construct • More accurate • Use online tools to get dynamic measurements • Explore conjectures • NCTM Technology Principle employed

Figure 5: Comparison of Geoboards and GeoGebra

Based on the cognitive development of children, it is recommended that parents and teachers challenge the child's abilities, but "not" present material or information that is too far beyond the child's level. It is also recommended that teachers use a wide variety of concrete experiences to help the child learn (e.g., use of manipulatives, working in groups to get experience seeing from another's perspective, field trips, etc). See Figure 6.

Recommendations from a Geometric Viewpoint

Geoboards	GeoGebra
Use as concrete activities in the early elementary grades to provide a foundation for more abstract thinking	Move to a more generalized abstract thinking in order to explore dynamic measurements and conjectures

Figure 6: Geoboard vs GeoGebra Comparison

Summary

So why should elementary math teachers use GeoGebra? First it is free to download and use via geogebra.org. Secondly, GeoGebra is a new dynamic teaching tool available in our schools today. Because it is user-friendly for students and teachers, GeoGebra is great way to connect the hands-on geoboards to virtual geoboards to something even more abstract. GeoGebra provides many resources and teaching tools at its wiki for educators at: http://www.geogebra.org/en/wiki/index.php/Main_Page. One of the best reasons for using GeoGebra is that it can even be used for primary-aged students. By using this simple dynamic tool, students find learning geometry, algebra, and measurement fun. PowerPoint and Data Files for GeoGebra as they relate to this presentation and paper can be accessed at**:** http://matharoundus.com/geogebra

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Fahlberg-Stojanovska, L, & Stojanovski, V. (2009). GeoGebra- freedom to explore and learn. *Teaching Mathematics and Its Applications: An International Journal of the IMA*, *28*(2), 49-54.

Furner, J. M., & Marinas, C. A. (2007). Geometry sketching software for elementary children: Easy as 1, 2, 3. *Eurasia Journal of Mathematics, Science & Technology Education,* 3 (1), 83-91.

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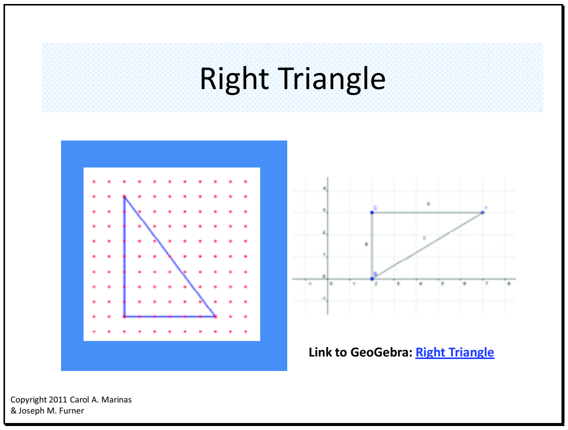
Reys, R E., Lindquist, M. M., Lambdin, D. V., Smith, N.L., & Suydam, M. N. (2009). *Helping children learn mathematics (9th Ed).* Boston, MA: John Wiley & Sons Publishing, Inc.

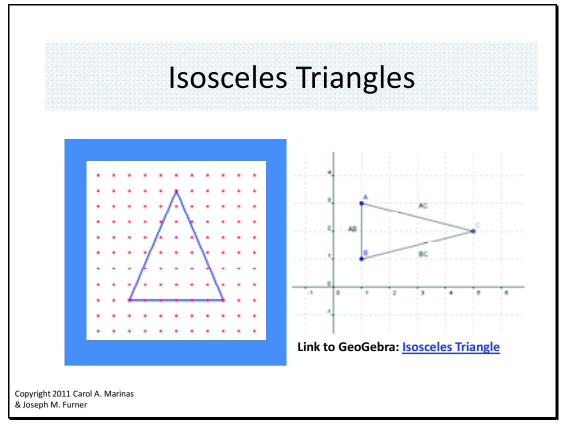
Rosen, D., & Hoffman, J. (2009). Integrating concrete and virtual manipulatives en early childhood mathematics in early childhood mathematics. *Young Children,* 64 (3), 26-33.

Scandrett, H. (2008). Using geoboards in primary mathematics: going…going…gone? *Australian Primary Mathematics Classroom,* 13 (2), 29-32.

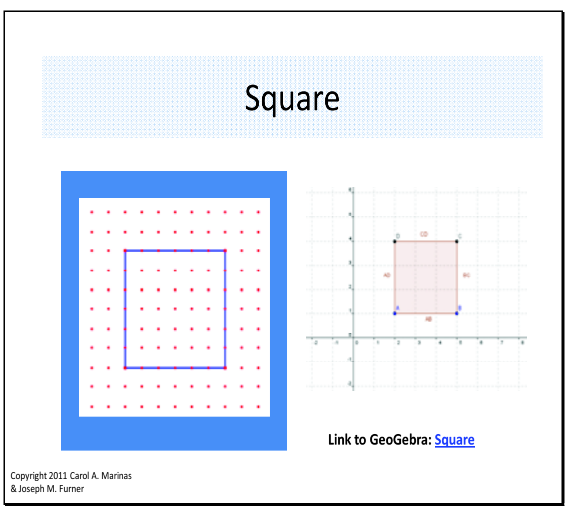
Appendix A: Sample GeoGebra Activities for Elementary Students



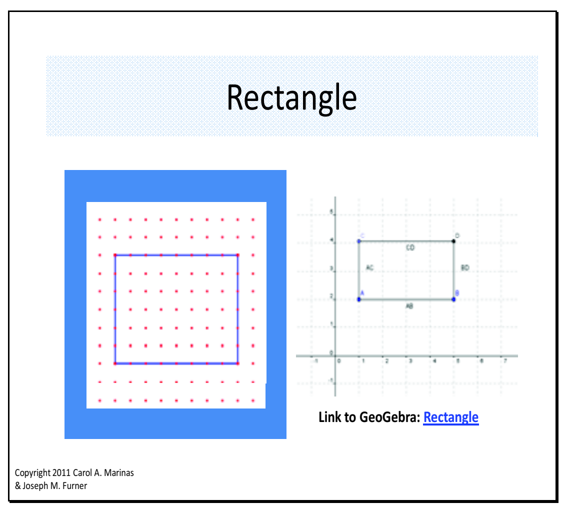
Students can explore measurement of right angles. Explore 3-4-5 triangle for Pythagorean Theorem.



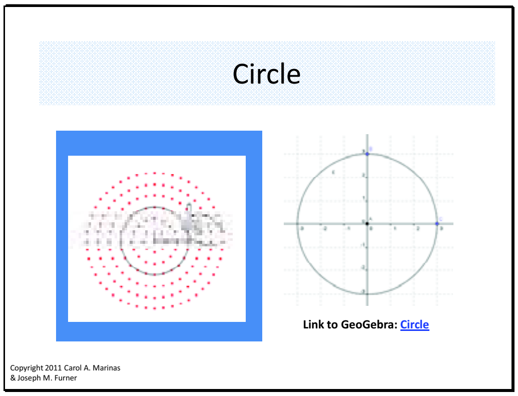
Students can see that 2 sides and 2 angles have the same measure.



Students can explore graphing ordered pairs on the coordinates to see all sides are equal.



Students can explore area and perimeter of rectangles.



Students can learn about radius, diameter, circumference and the relationship of the diameter to the circumference, π.

Appendix B: Virtual Manipulative Websites

National Library of Virtual Manipulatives: http://nlvm.usu.edu/en/nav/topic_t_3.html

National Council of Teachers of Mathematics:
http://www.nctm.org/standards/content.aspx?id=25007

Cut the Knot: http://www.nctm.org/standards/content.aspx?id=25007

Geoboard Resources: http://msteacher.org/epubs/math/QuickTakes/geoBoard.aspx

GeoGebra: geogebra.org

GeoGebra Wiki Forum: http://www.geogebra.org/en/wiki/index.php/Main_Page

GeoGebra Data Files: http://matharoundus.com/geogebra