AN OVERVIEW OF A DECADE OF WEBWORK USE IN PRECALCULUS AND CALCULUS

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At the University of Hartford, the Mathematics Department in the College of Arts and Sciences has found that an internet based, interactive, self-reporting homework system significantly increases how much homework students actually do, resulting in improved learning and attitude. Additionally, faculty-student interaction rises dramatically on email and in office hours. The department currently has a database of problems for Precalculus, Calculus I and Calculus II that allows all full-time and part-time faculty to offer WebWorK in their sections. Students who use WebWorK come to expect regular homework assignments and are responsible about completing them which translates to greater success as they progress through the entire calculus sequence. WebWorK transforms students from passive listeners to active learners. With improved student success comes increased student retention and interest in mathematics and the sciences. WebWorK also facilitates and improves teaching with its extended administrative component that allows an instructor to easily evaluate and track student performance, communicate readily with students via email, and gather data on class results.

WebWorK is currently used in our department by all instructors in all sections of the calculus sequence. Through individual efforts the system is also being introduced into a number of other courses including Linear Algebra, Differential Equations, and Advanced Engineering Mathematics. Homework sets are fairly uniform, ensuring consistency of topic coverage. The benefits are the same as for any online interactive homework system. Students receive instant feedback and tend to have increased email contact with their instructor. The built in automated deadline and associated grade motivates completion of each assignment. A hardcopy with correct answers serves as a study guide for tests. Jane Doe sets can be created with public access for additional practice sets or to generate test problems, particularly useful if multiple versions of a single question are needed. Class surveys and individual comments indicate that the majority of students find online homework convenient and helpful.

Although today’s students are computer savvy, they need help developing and pairing good study skills with the technology. Because the computer instantly checks for accuracy of an answer, there is a tendency to start randomly trying numbers when a solution doesn’t work. They need to be reminded that typing in all the numbers from 1 to 100 is not helping them learn. They also frequently procrastinate until the last minute and then send a barrage of messages to their instructor at midnight. Just as with written homework, it takes time for them to develop an ability to pace themselves when meeting a one week deadline for a homework set. With the technology there is a temptation to neglect to write anything down when doing homework. If a structure is suggested, such
as keeping a homework notebook, students are more likely to keep good notes. Study groups can safely be encouraged with little concern about sharing answers since problem values are randomized. There are occasional cases in which a student achieves a very high WeBWorK grade and manages only low scores on tests. This can indicate that the individual is getting too much assistance on homework. An effective way to address this is to have a discussion with the student and explain the relationship between homework and learning, making suggestions on alternative ways to seek out help on the material. Students also benefit from guidance in how to compose a clearly stated question on email because their communication skills are usually weak.

As a self-contained homework delivery system, WeBWorK is successful in improving student completion of assignments. However, there are a number of creative ways to use it that enhance its effectiveness in and out of the classroom. During lecture, referring to the instructor set for a few solved examples helps students become comfortable with the format. When a homework question is asked in class, the instructor can bring up that student’s set and address their particular problems. This motivates other students to ask more questions in class because they realize that they will get individualized help if they do. Working through one student’s question usually addresses the issues of most other students on that same problem. When tackling lengthy take-home projects that span two or more weeks, students have different rates of progress. If time is allotted to working on the project in class, using Student Progress data can assist in creating groups. The instructor can sort by current scores and assign groups of similar progress to work together or they can choose to cluster weaker students with a strong student in a tutoring role.

One feature available on the system is the ability to “rebuild” a set by deleting an individual student’s homework set and generating a new one that is identical except for the randomized variables. This allows a student to redo a set that has a low grade. The motivation for the student is that they can raise their WeBWorK average. The pedagogical advantage is that the student automatically reviews old material by redoing homework problems. There are a number of creative ways that this feature can be used. If a particular set has unusually low results, the set can be reassigned to the entire class. But there will always be a few students who have higher grades and will feel that this is unfair additional work. Since rebuilds can be done individually, offering a “Redo” set with a signup sheet is often the best alternative. This is feasible with small or medium size enrollment sections. The result is targeted studying, self-assigned by the students themselves. By having students make a commitment to the Redo with no option of retrieving their earlier score, they improve their scores with few exceptions. To increase studying before a test, opening up two or three sets related to the material covered essentially “tricks” students into additional studying before a test. They are willing and even eager to sign up because they view it as an opportunity to improve their scores. Frequently, students with high scores will sign up for a Redo set just for the chance to get a few extra points.

The single most significant advantage that WeBWorK currently has over the other major homework systems is that it allows faculty the ability to edit and write randomized
problems. Other commercial products provide a more attractive user interface and frequently integrate an online textbook and course management system, each of which is highly functional. However, to simply import textbook problems into a database that requests only the final answer raises issues of assessment accuracy if students have access to a Computer Algebra System (CAS), some of which are available free and online. The solution input for many online homework systems is usually a single answer at the end of the problem posed. Since the TI-89 and any other CAS can compute derivatives and integrals of functions, basic drill style problems such as those involving chain rule or u-substitution are better presented in a multi-step format. Breaking down a problem serves two purposes. It allows for partial credit and provides feedback at multiple stages as the student solves the problem. It also results in more accurate testing of skills and conceptual understanding because students are required to give more than the final answer which the calculator could easily provide with the push of a single button. In the case of an application such as a related rates problem that requires several steps, a student may have completed the majority of the problem correctly and still get the answer wrong due to a single miscalculation in the last step. WeBWorK allows instructors to edit and revise problems to allow for more in-depth assessment and increased learning through homework. Several examples of department written multi-step WeBWorK problems from Precalculus, Calculus I and Calculus II follow.

**WeBWorK Example 1. Calculus I: Chain Rule.**

<table>
<thead>
<tr>
<th>Entered</th>
<th>Answer Preview</th>
<th>Correct</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>sin(x)</td>
<td>sin(x)</td>
<td>sin(x)</td>
<td>correct</td>
</tr>
<tr>
<td>x^6</td>
<td>x^6</td>
<td>x^6</td>
<td>correct</td>
</tr>
<tr>
<td>cos(x^6)</td>
<td>cos(x^6)</td>
<td>cos(x^6)</td>
<td>correct</td>
</tr>
<tr>
<td>6*(x^5)</td>
<td>6*x^5</td>
<td>6*x^5</td>
<td>correct</td>
</tr>
<tr>
<td>cos(x^6)<em>6</em>(x^5)</td>
<td>cos(x^6)<em>6</em>(x^5)</td>
<td>cos(x^6)<em>6</em>(x^5)</td>
<td>correct</td>
</tr>
</tbody>
</table>

All of the answers above are correct.

(1 pt) M144L-library/Chap3_5/prob6.pg
The function \( h(x) = \sin(x^6) \) is differentiated using the chain rule.

If we let \( f(x) \) represent the outside function, then \( f(x) = \)

\( \sin(x) \)

If we let \( g(x) \) represent the inside function, then \( g(x) = \)

\( x^6 \)

\[ f'(g(x)) = \cos(x^6) \]

\[ g'(x) = 6x^5 \]

Now apply the chain rule to calculate the derivative of \( h \).

\[ h'(x) = \cos(x^6) * 6x^5 \]
WeBWorK Example 2. Calculus I & II: u-substitution.

<table>
<thead>
<tr>
<th>Entered</th>
<th>Answer Preview</th>
<th>Correct</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>cos(x)</td>
<td>cos(x)</td>
<td>cos(x)</td>
<td>correct</td>
</tr>
<tr>
<td>-[sin(x)]^2*dx</td>
<td>- (sin(x)) dx</td>
<td>-sin(x)d^2x</td>
<td>correct</td>
</tr>
<tr>
<td>u^2</td>
<td>u^2</td>
<td>u^2</td>
<td>correct</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>correct</td>
</tr>
<tr>
<td>(-1/3)*[(u^3)+C</td>
<td>(-1/3)u^3 + C</td>
<td>(-1/3)u^3 + C</td>
<td>correct</td>
</tr>
<tr>
<td>(-1/3)*[(cos(x))^3]+C</td>
<td>(-1/3)cos^3(x) + C</td>
<td>(-1/3)cos^3(x) + C</td>
<td>correct</td>
</tr>
</tbody>
</table>

All of the answers above are correct.

(1 pt) M144Library/Chap5_5/prob23.pg
Evaluate the indefinite integral \( \int \cos^2(x) \sin(x) dx \) by making a u-substitution.

\[ u = \cos(x) \quad \text{(Give a function of } x \text{ here.)} \]
\[ du = -\sin(x)dx \]

When you make the substitution, the integral becomes \( k \int g(u) du \) where

\[ g(u) = u^2 \quad \text{(Give a function of } u \text{ here.)} \]
\[ k = -1 \quad \text{(Give the constant that would be needed outside the integral.)} \]

Now do the antidifferentiation.
\[ k \int g(u) du = \frac{-1}{3}u^3 + C \quad \text{(Don't forget the } +C, \text{ "C" must be capital not lowercase.)} \]

Give the final result by substituting the function of } x \text{ back in.
\[ \int \cos^2(x) \sin(x) dx = \frac{-1}{3} \cos^3(x) + C \]
WeBWorK Example 3. Calculus II: Integration by Parts.

(1 pt) M145Library/Chap5_6/prob5parts.pg
Use integration by parts to evaluate the integral

\[ \int 4x \sin(x) \, dx \]

by filling in the steps below.

\[ u = \quad dv = \]
\[ du = \quad v = \]

Apply the integration by parts formula to get

\[ \int 4x \sin(x) \, dx = \quad - \int \]

Now do the second integration to get the final answer.

\[ \int 4x \sin(x) \, dx = \quad + C \]

WeBWorK Example 4. Precalculus: Quadratic Applications.

A rancher with 10000 ft of fencing wants to enclose a rectangular area and then divide it into 5 pens with 4 interior fences parallel to one side (W') of the rectangle (see the figure). The dimensions of the pen will be L (length) by W (width).

\[ \text{a. Find a function that models the total area of the pen in terms of } L \text{ and } W'. \]
\[ \text{Area} = \quad \frac{L \times W'}{\quad \text{feet}} \]

\[ \text{b. Set up an equation that expresses the 10000 ft of fencing in terms of } L \text{ and } W'. \]
\[ 10000 = \quad 2L + 6W' \]

\[ \text{c. What are the dimensions of the pen of greatest area?} \]
\[ \text{Width} = \quad 833.33 \quad \text{feet} \]
\[ \text{Length} = \quad 2500 \quad \text{feet} \]

\[ \text{d. Find the largest possible total area of the pen.} \]
\[ \text{Area} = \quad 833.33 \times 2500 \quad \text{square feet} \]
ACKNOWLEDGEMENTS. For their initial work in piloting WeBWorK, thanks are due to Dr. Ben Pollina & Dr. John Williams. Administrative support and constant encouragement on the part of Dr. Joel Kagan, Chair of the Department of Mathematics have kept this program thriving. Funding provided by grants received from both the University of Hartford’s Faculty Center for Learning and Development and the Connecticut Space Grant College Consortium have supported the author’s research in development and enhancement of the WeBWorK problems database. Dr. Ben Pollina generously provides all system maintenance including software upgrades, and is responsible for faculty training for the Department of Mathematics.