

# ONLINE CALCULUS – CONTENT, TOOLS, AND METHODS

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## Introduction

We present a Single Variable Calculus course that has been delivered purely online for the past three years at the University of Helsinki and partially online at Florida State University. We present the tools used to deliver the course as well as discuss its methodological and pedagogical design as a case study. The development of the methods and contents mainly took place during the European Commission funded WebALT project after which the company WebALT Inc. maintains and further develops them.

Current platforms and virtual learning environment integrate a number of web technologies that allow delivering rich learning materials such as video-lectures, slideshows, and interactive applets and exercises. The wide choice of materials and of ways of presenting them must however be narrowed according to the cognitive capabilities of the students and the goals set for learning, so that the focus stays on the subject matter and does not derail to the technologies themselves. In our experience, it is important that the environment and the materials themselves are designed to support collaborative learning, to track the learning process, and to adapt to the student's individual learning style.

## Format of the Calculus course

The online Calculus course is organized based on a weekly schedule comprising several activities. First of all, before the scheduled weekly discussion hour, students have the task to watch a lecture that introduces new concepts, their applications, and sample problems. Then, the discussion session relies on the questions and issues that have arisen in the forum discussion threads meant to collect feedback from the students. Assignments are also given as a task for the student to solve and present to the class during a synchronous online exercise session. In addition to these, unlimited practise with online interactive mathematics problems is available to the students during the whole course.

As a learning platform the open source Moodle virtual learning environment was chosen. Moodle is easy to use and flexible and moreover, extensible with a wide range of plug-ins

developed by the active user community. Several plug-ins especially for mathematics education have also been developed. All the activities and course materials were integrated into the calculus course home page within Moodle.

The course emphasizes collaborative learning in a number of ways. Real-time voice discussions are encouraged and facilitated during the virtual discussion hour that takes place within a web conferencing system. Asynchronously, students are expected to start an on-line discussion thread in the course forum where they can post their comments, notes, and questions for other students to answer and comment further.

To facilitate a meaningful discussion within a mathematics course, an easy to use method for inputting mathematical formulas into forum posts is necessary. Moodle supports so called input filters that allow editing and displaying different content types. For example, Moodle provides a LaTeX input filter by which users can input LaTeX formulas between double dollar signs, e.g. `$$ \int_0^1 \frac{2x}{1+x^4} dx $$` and then this formula is displayed as a png image inside the text. This is convenient way of inputting formulas but first year students generally do not know LaTeX commands. We utilized the Wiris input filter that incorporates the Wiris 2D formula editor into the Moodle text editor. It provides a palette of usual symbols and therefore is ready to be used by everybody.

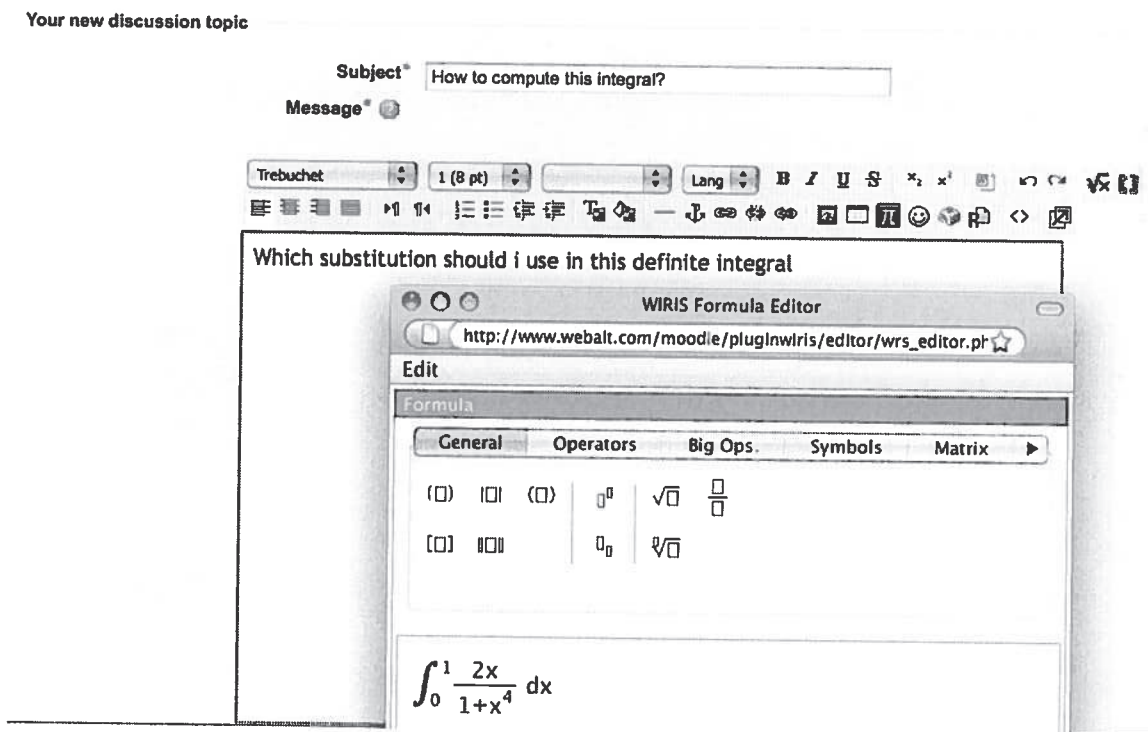


Figure 1. Wiris formula editor in Moodle

### Practicing with interactive exercises

One of the most important benefits of a technology enhanced learning environment is its ability to provide immediate feedback to students. This is needed for students to gain un-

Understanding of their learning process. For this reason, the course provides a collection of web-based interactive mathematics exercises that have proven to be useful in this respect. As a platform of authoring and delivering these exercises we use the MapleTA online assessment system. While practicing with the exercises, students gain confidence about their understanding and skills. Some of the parameters in the exercises change each time it is taken which motivates practicing. In fact, the exercises can be viewed as computer programs utilizing the Maple language, which generate the exercises, feedback and evaluation for the correctness of the user's answer. Moreover, students learn the mathematical concepts and techniques from the system-generated detailed solutions. Some problems incorporate links to a video explaining a solution to a problem of similar type.

The image shows a screenshot of a MapleTA interface. On the left, there is a navigation menu with 'Calculus, Limits' selected. Below it, 'Question 1: (1 point)' is displayed. The question asks to evaluate the limit:  $\lim_{t \rightarrow -2} \left( \frac{3t^2 - 9t + 9}{-5t^2 - 8t + 7} \right)$ . A note states: 'NOTE: If a limit does not exist (or is infinite), you can enter your answer as a solved problem of this type.' Below the note is an input field and links for 'Relevant lecture', 'Plot', 'Help', 'Preview', 'Hint 1', and 'Hint 2'. Below this is 'Question 2: (1 point)' which asks to evaluate the limit:  $\lim_{x \rightarrow -2} \left( \frac{3x^2 - x - 3 + \sqrt{-x^2 - 2x + 3}}{-4x^2 + 3 + \sqrt{-2x}} \right)$ . On the right, a video player is embedded, titled 'How to Compute Limits (2)'. The video content shows an example:  $\lim_{x \rightarrow 1} \frac{x-1}{1+x^2} = \frac{1-1}{1+1^2} = 0$ . The video player has a progress bar and size options (Small, Medium, Large).

Figure 2. A calculus exercise with a link to video with solution

Occasionally these exercises were used as an immediate quiz during the online discussion session. After explaining a concept, the instructor asked the students to take a quiz consisting of two to three questions about the material just presented. This gives immediate feedback both to the students and to the instructor on how well the concept was understood and therefore provides a possibility to correct mistakes right in the beginning. Also we noticed that students tend to be more attentive when they know there will be a quiz afterwards.

## Support materials

The support materials for the weekly activities are made available well in advance and in a variety of formats, e.g. slideshows, handouts for printing, and videos with voice narration. The solved exercises are also provided in these formats together with the unsolved assignments. This allows students to self-organize study according to their learning style, for instance by tackling the unsolved problems first and checking the theory lectures later. Online availability also means that learning can occur at anytime, anyplace, and

moreover, the variety of formats accommodates individual preferences (listening versus reading).

The videos are linked from the Moodle course but they are also available as podcasts. When a student subscribes to the content, the lectures are automatically delivered to the viewing device such as an iPod touch. The videos are designed to be viewable also with smaller screens which means that the font size is fairly large and there is not too much content per page. Also the material is split into fairly small pieces explaining a specific concept or method thus capturing student's attention long enough. For example after viewing a presentation about theory the student can continue with presentations explaining applications or solved problems.

### **Feedback from students**

We present here some of the feedback from students who attended the Fall 2008 version of the course. Moodle provides convenient questionnaire facility by which we gathered feedback concerning the various tools used in the course. We select here feedback that helps us to further develop and improve the methods and content used in the course.

About the learning material one student commented "Podcasts are an excellent alternative to lectures, if I missed something I can always rewind and watch again", other said: "I've found the podcasts good and suitable for learning but sometimes the more difficult concepts are explained in a way that I don't understand. I've also found help from Wikipedia and other schools' materials on the web". This shows that today's students are capable of finding extra materials from the web that match best their learning style.

The MapleTA exercises were considered a valuable aid to learning especially with the feedback solutions. Some problems were caused by the specific syntax by which the formulas had to be entered but in general this is learned with a little practice. One student commented: "The exercises have been easy to use and very useful, without them there would not be enough work to do in order to learn."

The discussion forum did not turn out to be used as much as we expected. One student commented: "Trying to communicate my problems on math in a written form was difficult. I find it easier to talk directly to a person when I get continuous feedback whether my question is understood". We can note here that the ability of expressing mathematical ideas and questions in writing is a useful skill that also requires practicing. On the other hand, a student said: "The forum is a really good idea. Since one listens to the lectures at own time, without the forum the unclear issues would not be clarified – one tends to forget questions if one cannot ask immediately. Also the forum motivated to follow the lectures regularly".

The virtual course was seen as a reasonable alternative to traditional courses especially among those students that had a job or had a busy schedule of lectures otherwise. Most students said that they learn mathematics as well in this course as in a traditional course. On the other hand, the social aspect of the course can be improved, one student commented: "In the beginning, the lecturer talked about building a social network around this

course. There is a place for improvement here since interaction with other students did not really take place, at least in the extent as in traditional courses. Common online chat sessions or a common discussion forum could help this”.

## **Conclusions**

Over the past years while offering this course we have become confident that virtual online mathematics courses are useful and learning happens as well as in traditional courses. Such a course must be organized well, must provide a rich and versatile set of high quality learning materials and support interaction with the learner. Good communication channels are needed for students to ask questions and discuss about the content. The benefits of using ICT in education range from the obvious time and place independent learning to the immediate feedback that the online assessment provides. ICT has the potential to improve education and make quality instruction available to everybody.

Interaction is perhaps the most challenging feature of virtual learning: interaction between learner and the content, but also interaction between the learners themselves and learners with instructors. Modern web technologies when best put in practice can provide rich interaction capabilities to virtual education.

## **Resources**

WebALT: <http://www.webalt.com>

Moodle: <http://moodle.org>

Wiris: <http://www.wiris.com/>