USING SPREADSHEETS AS PROBLEM-POSING ENVIRONMENTS IN ELEMENTARY TEACHER EDUCATION

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Abstract

This paper shows how the combination of computational and manipulative features of an electronic spreadsheet can be put to work in providing a problem-posing environment in the context of the mathematical preparation of elementary teachers. It argues for the importance of training the teachers in formulating problems through the use of technology, something that can be viewed as a research-like experience in mathematics pedagogy. The notion of numerical and contextual coherency in problem posing is discussed.

Problem posing has long been recognized as an important pedagogical tool in the teaching of mathematics (Brown & Walter, 1983). The advent of technology into the classroom brought about the recognition of the potential of computing to enhance this tool (Kilpatrick, 1987). Just as discoveries in mathematics can be motivated by technology (Gleick, 1987), the appropriate use of computer applications can inform problem-posing activities across grades. Because any changes in mathematics pedagogy must be feasible from the very outset in the chain of children's educational experiences, the preparation of prospective elementary teachers in computer-enabled problem-posing techniques is an important educational task.

Indeed, in developing professional standards for teachers, the National Council of Teachers of Mathematics (1991) suggested:

> *There are a variety of ways technology may be used to enhance and extend mathematics learning and teaching. By far the most promising are in the areas of problem posing and problem solving in activities that permit students to design their own explorations and create their own mathematics* (p. 134).

Although the *Standards* did not provide specific examples in support of this powerful statement, the very emphasis on technology-enabled problem posing was at that time due to the advances in the development of dynamic geometry environments within which multiple examples can be explored and, as a result, new hypotheses (or, alternatively, problems) can be formulated.

Whereas geometry can be viewed as a traditional context for posing problem with technology (e.g., Yerushalmy, Chazan, & Gordon, 1993), recent advances in the use of spreadsheets in mathematics education (Baker & Sugden, 2003) enable other areas of mathematics to be explored from a problem-posing perspective in a technological paradigm. Already at the elementary level, the appropriate use of a spreadsheet makes it possible to turn a routine problem into a mathematical investigation. Through such an investigation, the numbers involved in such a problem become parameters that can be altered and tested in a problem-solving situation and then chosen to signify the completion of the problem-posing phase of the activity. This paper extends research and development activities related to the use of spreadsheets in problem posing by prospective secondary mathematics teachers (Abramovich & Brouwer, 2003; Abramovich & Norton, 2006) to include prospective elementary teachers. It suggests that spreadsheet-enabled mathematical problem posing must be presented to the teachers as a two-phase process: (i) posing a problem and (ii) ensuring its grade-appropriate solvability by finding the right balance between the ease and the challenge.

Open-ended pedagogy and problem posing

Research has been emphasizing the potential of an open-ended approach to the teaching of mathematics for some 30 years (e.g., Shimada, 1977; Becker & Selter, 1996). This approach challenges the traditional pedagogy of "only one correct answer" and questions its effectiveness. Instead, it focuses on uncovering "new" mathematics in familiar contexts, and creating problematic situations characterized by the multiplicity of answers. Problem-posing activities enhanced by technology have great potential to introduce this kind of mathematics pedagogy to prospective elementary teachers and their students alike.

As an example, consider the following problem with hidden open-ended structure adapted from New York State Testing Program (1998): *John has two quarters, one dime, two nickels, and two pennies, while Sarah has a quarter, a dime, and a nickel. Which coins could John give Sarah so that they both have the same amount of money?*

In order to help prospective teachers formulate similar problems (with the goal to enrich existing curriculum materials), the authors designed a spreadsheet-based environment with multiple worksheets. This environment allows the teachers to solve a problem by using grade-appropriate strategies based on the combination of trial-and-error computation and physical manipulation of objects (coins) on a computer screen. Such pedagogical approach turns the original problem into an open-ended one whose numerical structure is flexible and can be altered and tested during the problem-solving phase of a problem-posing activity.

Figures 1 and 2 represent, respectively, two types of worksheets – computational and manipulative – by using which a money-sharing problem can be posed and solved. While the former type includes a single worksheet associated with numerical structure of the problem, the latter type includes multiple worksheets designed to deal with its contextual structure. The contextual structure of the problem determines money sharing on a physical level, given specific sets of coins and rules of action in this process. For example, the two sets of coins pictured in Figure 2 ({25, 25, 10, 10, 1, 1} and {25, 10, 5}) do not allow one to share money without exchanging coins. Therefore, the contextual part of problem posing is responsible for its hidden complexity. In addition to being used with prospective elementary teachers, this environment can be recommended to be used by young children as long as three didactical objectives underlie it: embedding mathematical action into situated arithmetic (Lave, 1988), providing experience with the multiplicity of answers (Becker & Selter, 1996), and developing money concept through one-to-one correspondence (Piaget, 1961). More details about the third objective can be found elsewhere (Abramovich & Cho, 2006).

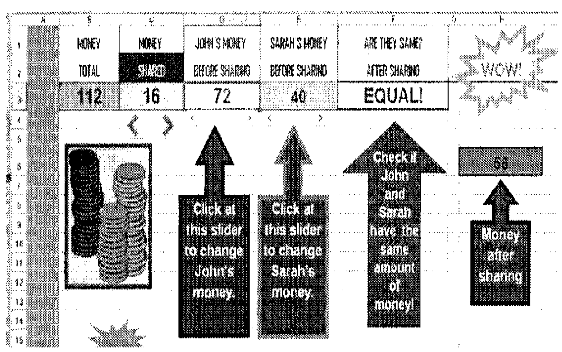

Figure 1. Computational Worksheet

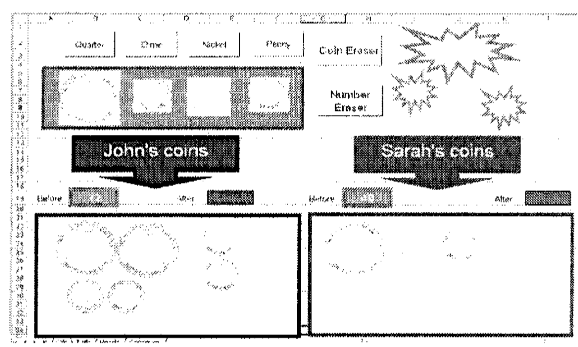

Figure 2. Manipulative Worksheet

Two approaches to the use of technology in problem posing

A traditional approach to the use of technology in problem posing consists of asking a "what-if" question and then taking advantage of technology as a medium for exploring multiple examples. This approach, described, for example, by Knuth (2002) in the context of geometry, does not really require technology for posing a problem. Rather, technology is used here to *solve* a problem (or a family of problems) once it has been posed; in other words, the solvability of a problem depends on a computational medium.

Another approach (emphasized in this paper) to the use of technology in problem posing is to use computing in developing data that ensures the solvability of a new problem in the absence of technology. To clarify, consider the following two problems one of which is a technology-enabled extension of the other.

Original problem. *The sum of digits of all page numbers in Amy's new book equals 51. How many pages are in the book?*

Extended problem. *The sum of digits of all page numbers in Amy's new book equals 73. How many pages are in the book?*

Note that in the extended problem a very specific "what-if" question is asked: What if 51 is replaced by 73? One can discover that although between 51 and 73 there are other numbers (not all!) that can represent a sum of digits of all consecutive numbers starting from one, 73 is the smallest number that satisfies this problem contextually. In other words, whereas both problems can be solved without using technology, one needs it to generate data that ensures problem's solvability.

By using a spreadsheet, a prospective teacher can generate problems of that type without much difficulty. Such a spreadsheet can be developed by the instructor. This leads to another issue associated with the use of technology that will be discussed in the next section.

Numerical and contextual coherency in problem posing

What is a book in terms of the number of pages? How many pages may a book have? To answer these questions, note that a book is paginated by a series of consecutive numbers starting from one and comprised of four-page sheets. Such simple numerical insight into the context suggests that problem posing cannot be adequately understood without attending to the notion of a problem's numerical and contextual coherency. Indeed, an arbitrary integer cannot represent the sum of digits of all consecutive integers starting from one, let alone of all page numbers in a book. In such a way, the above-mentioned extended problem could not be posed correctly without exploring its data's numerical and contextual coherency. By just changing the sum of digits of all page numbers in a book, one can develop an ill-posed problem. In other words, any problem may have structural limitations on which problem posing depends. Such limitations can be explored computationally by using a spreadsheet. Figure 3 portrays such a spreadsheet. It shows that the correctness of Amy's new book problem is characterized by a pair of numbers; these are (12, 51) and (16, 73) for the original and extended problems, respectively. One can formulate a new problem by trying to explore the relation between the elements of such pairs. This, however, is beyond the scope of activities appropriate for prospective elementary teachers.

◇	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	page numbers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	sum of digits of page numbers	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9	10	2
3	sum of digits of all page numbers	1	3	6	10	15	21	28	36	45	46	48	51	55	60	66	73	81	90	100	102

Figure 3. Spreadsheet for Amy's new book

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