TEACHING MATHEMATICAL WRITING ELECTRONICALLY

Carl R. Spitznagel
Department of Mathematics & Computer Science
John Carroll University
Cleveland, OH 44118
Phone 216-397-4351
Fax 216-397-3033
spitz@jcu.edu

Introduction

Many undergraduate mathematics programs include a course that is designed to be the students' first experience with abstract mathematical thought and proof. At John Carroll University this comes (for most math majors) in the first semester of the sophomore year. Although the title and content of the course are *Introduction to Abstract Algebra*, the fundamental purposes of the course are to bridge the gap between algorithmic applications of mathematics (calculus) and higher-level abstraction, to build a foundation for later courses, and to teach the students the art of crafting and writing simple proofs.

When I was scheduled to teach this course in fall, 2004, I watched in horror as the preregistration enrollment grew to over 30. I would, on a daily basis, have to read and comment on thirty faltering attempts at mathematical writing—multiplied by the number of problems assigned! But then, while sitting through a rather boring talk at a conference later that summer, and fretting over my impending doom, an idea struck me: why not encourage (or perhaps *force*) the students to present only their *very best work*? This would mean that each student would turn in *less* work, so my grading burden would be lightened. But how could they possibly learn to write mathematics well, without the benefit of repeated feedback? Could "less" possibly be "more"?

I had already taken an interest in putting mathematics on the web, and had become aware that Blackboard Inc. had contracted with Design Science Inc. to incorporate the WebEQ™ mathematical authoring system into the Blackboard Learning System™. And since John Carroll University was already encouraging faculty to utilize Blackboard, it seemed a natural step to use a Blackboard "Discussion Board" as a presentation medium for student work. I could have students post their solutions, using appropriate mathematical symbolism, for other class members to read. And although not every student would turn in every problem, every student would have the benefit of seeing what others were doing right (and wrong) in their proofs.

The System

1. Computational problems were assigned on an "honor" basis. I assigned only problems for which the students could check their answers in the book, and I did

not collect those problems.

2. For each theoretical problem,

   - I posted the statement of the problem on Blackboard, using the built-in WebEQ editor for including mathematical expressions. Within the Discussion Board, I started a new "Forum" for each problem. Using a different forum for each problem assures that all postings pertaining to a given problem will appear together, in the correct order. I chose a naming convention that made it easy for students to locate the forum for any given problem. For example, the forum for problem 18 on page 135 of the book was titled "p135n18."

   - One student was designated to be the official "problem solver." This student was to post a solution to the problem by the specified time, using WebEQ to include mathematical expressions in the solution where appropriate.

   - Two students were designated to be official "readers." By the specified time, these students were each required to post a critique of the posted solution, addressing issues of both substance and style, and using WebEQ to show corrections to the posted solution wherever needed.

   - After the critiques were posted, I read and commented on all three postings—the original solution and the two critiques. I assigned grades to all three postings, with a 10-point maximum for the solution, and 5-point maximum for each critique.

   - All students in the class were expected to at least attempt to solve each problem on their own, and then to read and learn from the Blackboard discussion on the problems.

   - The solver and the two readers each then re-wrote the problem and solution, using Word and MathType™, another product of Design Science, to turn in for a final reading and an additional 5 points of credit. I chose MathType as the equation editor for final write-ups because it works seamlessly with Word, produces mathematical typesetting of very high quality, and yet has a very gentle learning curve.

3. As in all of my classes, students were allowed and encouraged to see me for hints or suggestions, at any stage in the problem-solving and presentation process.

4. I would typically set 12:00 noon on a specified day as the deadline for electronic posting of solutions. The critiques were typically due to be posted by noon of the following day, and I made every effort to post my own comments by that evening. Final write-ups were then due several days later, to give the students involved an additional opportunity to see me for any clarifications. Because severe penalties were levied for late postings, there were very few problems with tardiness.

5. Roles were rotated so that experience as a solver and experience as a reader were equitably distributed among the students.

An Example

Figure 1 shows a portion of the postings for one particular assigned problem. Figure 2 shows the final solution submitted by one of the students. This example comes from approximately the middle of the course, and is the work of students whose final grades were among the middle fifty percent of the class. It therefore represents what might be considered typical of the quality of work the students produced in this class.

Forum: p120n27a
Date: Fri Nov 05 2004 09:54
Author:
Subject: Problem statement

Times Read: 54

(Remove)

Let $G = \{ [a] \mid [a] \neq [0] \} \subseteq \mathbb{Z}_n$. Prove that G is a group with respect to multiplication in $\mathbb{Z}_n$ if and only if n is a prime.

(Reply)

Forum: p120n27a
Date: Mon Nov 08 2004 22:31
Author:
Subject: Solution

Times Read: 47

(Remove)

First, suppose that n is prime. Then we must show that the following hold true for the group

$G = \{ [a] \mid [a] \neq [0] \} \subseteq \mathbb{Z}_n$.

1. Associativity holds for G.

2. There is an identity element for G.

3. Every $[a] \in G$ has an inverse.

Proof of 1.

Theorem 2.27 proves that multiplication is associative on $\mathbb{Z}_n$.

Proof 2.

For $n > 1$, [1][x]=[x]=[x][1] for all $[x] \in \mathbb{Z}_n$.

Proof 3.

Suppose n is prime, since $[0] \neq [a]$ we know that $(a, n)=1$. Then there exists classes $[m]$ and $[b]$ such that $[a][m] + [n][b] = [1]$. So, $[a][m] = [1] - [n][b]$. Then $[a][m] = [1]$ (mod n). Thus $[a]^{-1} = [m]$.

In order to finish the proof, we must show that if G is a group, then n is prime.

Suppose n is not prime. Since n is not prime, we can write $n = [a][b]$, where $[0]<[a]<n$ and $[0]<[b]<n$. So $[a][b] = [0]$. Then

$[a]^{-1}[a][b][b]^{-1} = [a]^{-1}[0][b]^{-1}$. Then [1][1] = [0]. Thus [1] = [0] which is clearly a contradiction. Hence n must be prime.

(Reply)

Figure 1: Sample Blackboard Posting

MT 341. 51
Homework for Nov. 12
by

Problem p120n27a

Let $G = \{[a] \mid [a] \neq 0\} \subseteq \mathbb{Z}_n$. Prove that G is a group with respect to multiplication in $\mathbb{Z}_n$ if and only if n is a prime.

Solution:

First suppose n is prime. Then we must show that the following conditions hold true for the group $G = \{[a] \mid [a] \neq 0\} \subseteq \mathbb{Z}_n$.

1) To show that multiplication is a binary operation on G, we must show that for $[a] \neq [0]$ and $[b] \neq [0]$, $[ab] \neq [0]$. Supposing that n is a prime, we know that $[n] \neq [0]$. Also, by the UFT we can write n as a product of primes, say $n = ab$, where $1 < a < n$ and $1 < b < n$.
So we have $[a] \cdot [b] = [ab] = [n] \neq [0]$. Thus if $[a] \neq [0]$ and $[b] \neq [0]$, then $[ab] \neq [0]$. Then multiplication is indeed a binary operation on G.

2) By part b in Theorem 2.27, multiplication is associative in $\mathbb{Z}_n$. So, associativity holds for G.

3) By part c in Theorem 2.27, $\mathbb{Z}_n$ has a multiplicative identity of $[1]$. So there is an identity element for G, such that for any $[a] \in G$, $[1] \cdot [a] = [a] = [a] \cdot [1]$. Thus $[1]$ is an identity element for G.

4) Lastly, we must show that every $[a] \in G$ has an inverse. Suppose n is prime. Then we know $(a, n) = 1$ because of the proof of Theorem 2.28 done in class. Then, with a relatively prime to n, we use Corollary 2.29 to say that every non-zero element of $\mathbb{Z}_n$ has a multiplicative inverse. Thus every $[a] \in G$ has an inverse.

Conversely, supposing that G is a group with n NOT prime, we get $[a] \neq [0]$ for $[a] \in \mathbb{Z}_n$, by homework problem p90n21, but $[a] \cdot [b] = 0$. This says that G is NOT closed under multiplication, giving us a contradiction. So n must be prime.

Thus if $G = \{[a] \mid [a] \neq 0\} \subseteq \mathbb{Z}_n$, then G is a group with respect to multiplication in $\mathbb{Z}_n$ if and only if n is a prime.

Figure 2: Sample Final Write-up

Student Reactions

Overall, student reactions to this system were very positive. The one concern that I had, prior to implementing the system, was that students would object to having their names associated with their work, for all the students in the class to see. And although I justified this to myself with the thought that it is really no different than calling on students in a discussion-based class, I was very pleased that, when asked on a course evaluation form to name three things that they did not like about the homework system, not a single student cited concerns about having their work scrutinized by the class.

The following comments are typical of the students' reactions to the electronic homework system:

- "The Blackboard system has helped out with my mathematical writing skills tremendously, before I started this course I had no idea how to write with a good mathematical style and I feel the critiques and comments have helped out a lot with that."
- "Working with WebEQ was not always fun."
- "I benefited from seeing other students' approaches to the problems. If I was stuck, the posted solutions gave me a place to start."
- "I think seeing in writing other students' attempts at mathematical writing, as well as the professor's critiques, was very helpful in learning to write correctly."
- "Since there weren't a lot of assignments there was a pressure to do well on your assignments."
- "I was able to read the professor's comments on the proofs that I worked on, as well as the proofs solved by other students.... I really liked all of the feedback that we got after doing the assignments."
- "The instructor was able to give in-depth comments for each problem."
- "I liked critiques by fellow students to understand if my solution was easy to understand."
- "It took a lot of discipline to read all the postings."
- "Blackboard allows us to have feedback on all the problems and in greater detail."
- "Posting problems and then turning in a final copy makes you more conscientious [sic] of your writing."

- "I learned how to grade or critique others, which is helpful since I want to be a teacher."

- "The homework system allowed the students to share ideas and communicate much easier."

Observations and Conclusion

With this system I found that I was able to emphasize the *writing* of mathematics more than I had in previous offerings of this course. I was able to provide much more extensive feedback on the writing of each solution than I could have, had I been reading and grading thirty students' efforts on each problem—and yet every student was able to benefit from seeing the work and mistakes of students on every problem.

There was greater student accountability, for two reasons. First, each student had relatively little of his or her own work formally evaluated, and so there was greater motivation to put forth a good effort on those problems. And second, every student in the class would be reading that work and seeing how it was critiqued and evaluated. As a result, the students tended to take this work very seriously; very few were tardy or overly sloppy with their posted solutions or critiques.

Because the students submitted their solutions and critiques electronically, the "collection" of homework could be asynchronous with the class. Although I chose to set the same deadline for all problems in a given section of the text, it was not necessary to make that deadline coincide with class meetings.

Although I did not have to suffer the boredom and frustration so often encountered when grading a large number of attempts to write a given proof, I found that my total time required by this system was probably about the same as if I had been grading individual solutions. With a web posting, it is not possible to simply circle an incorrect statement and write a short correction to it—so instead it was necessary to write my comments very carefully and clearly, so that the solver, the readers and all of the students in the class would understand those comments. In general, my comments on the posted solutions and critiques were far more complete than would have been possible with the traditional paper homework approach. This system also allowed me to afford greater attention to the students' writing and presentation style than had ever been possible in the past.

Although most of my students had had prior experience with Blackboard, I did have to devote one whole class to teaching the students to use the Discussion Board feature, and to use the WebEQ editor for inserting mathematical expressions in their postings. I also took that opportunity to explain by examples what I was expecting in their solutions and critiques. I also used some time during that one class period to teach the students to use MathType, which they needed for their final write-ups.

Of course, some students performed better than others. Some recognized the great learning opportunity afforded by the ability to read the work of their fellow students and see how it was critiqued, and were quite conscientious about reading the full discussion on every problem. Predictably, however, some others saw the system as an opportunity to try to get by with less effort. Since Blackboard tracks the frequency of user access to the Discussion Board, it was possible for me to get a good idea of which students were, in fact, reading the discussions as they were expected to. Needless to say, there was a strong relation between time spent in the Discussion Board and performance in the course. Although it is pure conjecture, I feel fairly certain that the students who did not take this component of the course very seriously were, for the most part, the same ones who would have turned in sloppy and incomplete homework papers under a traditional paper homework scheme. However, even those marginal students were able to feel the pride of producing correct and polished mathematical proofs, given the benefit of multiple critiques and the expectation that their final work on a problem would, in fact, be quite good.

In the future I will certainly continue to use this approach to teaching mathematical writing in the *Introduction to Abstract Algebra* course (our "bridge" course). I highly recommend this system for any course in which the focus is on learning to write mathematics in the context of sets, mappings, algebraic structures or analysis. Because of the complexity of mathematical notation in some other areas, however—such as linear algebra, discrete mathematics and geometry—I would probably choose to adopt some other method of teaching mathematical writing.