DEVELOPING PRECALCULUS CONCEPTS IN CONTEXT

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This paper discusses labs and projects based on real-world contexts that are useful in motivating and developing mathematical topics covered in Precalculus. The sample lab, project, and exploration contained in this paper are from *Precalculus: Concepts in Context* [1], a combined text and labs-projects-explorations manual. *Precalculus: Concepts in Context* is written for students who have studied, but not necessarily mastered, the equivalent of two years of high-school algebra and a year of geometry. The combination text/manual is based on four pedagogical principles:

- Writing about mathematics deepens understanding.
- Student exploration is at least as valuable as teacher explanation.
- Collaboration promotes genuine learning.
- Graphing technology has a role in the development of mathematical concepts.

Labs are designed for group collaboration, deal with a single context and problem(s) related to that context, and culminate in a single written report submitted by the group. Projects consist of a series of questions related to a single context. These questions develop and/or reinforce a specific mathematical topic. Projects are appropriate for small group in-class work, independent work, or take home exam/quiz questions. Explorations are mathematical investigations that require use of graphing technology.

At Eastern Connecticut State University (ECSU) Precalculus is a four-credit course with one 50-minute period each week for student-centered work on labs, projects, or explorations. Work on these assignments is generally started during this period and completed as homework. This paper presents descriptions of a lab (*Turtles*), exploration (*Copycats*), and project (*Fuels Rush In*).

Sample Lab: *Turtles*

The lab *Turtles* focuses on the plight of the endangered Kemp's ridley sea turtle. Students examine population data from three different time periods, 1947-1986, 1978 – 1985, and 1988 – 1997. Based on these data, they develop models for predicting population size and make projections about the turtle's endangered status.

Each lab begins with a preparation section that students complete prior to the lab period. In the preparation for *Turtles*, students discover that y versus x is exponential if and only if $\log(y)$ versus x is linear. In addition they learn how to transform models of the form $\log(y) = b + mx$ into exponential models of the form $y = a \cdot c^x$.

Alarming Decline, 1947 - 1986
The most dramatic change in population for the Kemp's ridley sea turtle occurred over the time period from 1947 to 1986. The estimates for the number of nesting females on a single beach at Rancho Nuevo in Tamaulipas, Mexico where 95% of the nesting occurs appear in Figure 1.

Year	x	Nesting Females, y	log(y)
1947	0	40,000+	
1968		5,000	
1970		2,500	
1974		1,200	
1986		621	

Figure 1. Estimates of the nesting female population at Rancho Nuevo.

A scatter plot of log(y) versus x appears approximately linear; linear regression produces the model $\log(y) = 4.578 - 0.0483x$ (See Figure 2.). Transforming this relationship into an exponential model yields $y = 37844(.8947)^x$ (See Figure 3.).

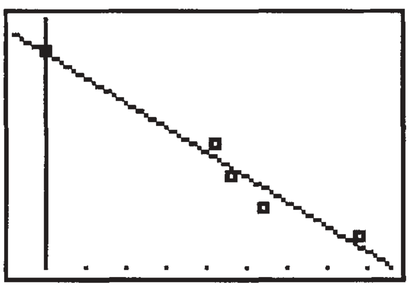

Figure 2. Scatter plot of log(y) vs. x.

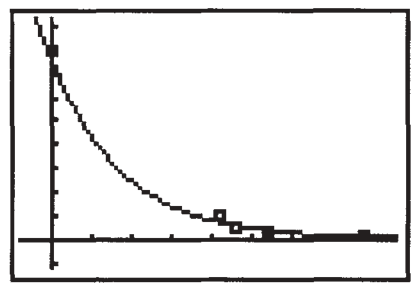

Figure 3. Scatter plot of y vs. x.

The exponential model represents an average annual decline of 10.5%, one of the most drastic population declines recorded for any animal. Based on this model, students predict the year that the Kemp's ridley turtle would, for all practical purposes be extinct. As a first attempt, many students try to solve the equation $37844(.8947)^x = 0$. After further thought, students often decide that the Kemp's ridley turtles will be extinct for all practical purposes when the number of nesting females is less than 1, which according to this model will occur by 2042.

Gradual Losses, 1978 - 1985
Because of this dire prediction, in 1978 a binational Kemp's Ridley Working Group was formed, comprised of Mexican and U.S. university researchers and several agencies. Their efforts resulted in increased protection at the nesting beach, promotion of the development of a second nesting beach, and an experimental "head start" program in which hatchlings were raised in captivity for their first year to decrease infant mortality. Despite these efforts, the Kemp's ridley population continued to decline. Dr. Rene Marquez (a scientist at Instituto Nacional de la Pesca) proposed the model $\log(N(x)) = 2.89 - 0.0195x$, where $N(x)$ represents the number of nesting females and x

represents the years since 1997. Students transform Dr. Marquez's model into an exponential model, determine the average annual percentage rate of decline, and again project the year in which the Kemp's ridley turtle would, for all practical purposes, become extinct.

A Vigorous Comback, 1988 – 1997.
With the Kemp's ridley turtle population still in decline, environmentalists turned their attention to another major cause of sea turtle mortality: drowning in the nets of shrimp trawlers. At first (1983), use of turtle excluder devices (TEDs) were voluntary but by 1994 their use became mandatory. Improvement in nest counts, n, from 1988 to 1997 appears in Figure 4.

Year	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
n	842	878	992	1155	1286	1229	1560	1943	2080	2387

Figure 4. Nest counts from a variety of sources.

Students fit an exponential model to these data and determine the annual percentage rate of increase for the number of nests per year. The previous two models, however, described population rather than number of nests. In order to build a population model based on these data, an estimate of the average number of nests per female is needed. However, there is some controversy over estimates of the average number of nests per female (from 1.3 to 2.3 nests per female). Students discover that this discrepancy changes only the constant multiplier in the exponential models and not the growth factor. Hence, the projected annual percentage rate of increase in population is unaffected by differences in the estimates of average number of nests per female.

Each lab culminates in a written report submitted by the group. The description for the written report for the *Turtles* lab appears below.

> Use mathematics to tell a story of the Kemp's ridley sea turtles. What is the purpose of using the logs of the data? Explain (that is, justify mathematically) why graphing $\log(y)$ versus x and getting a linear pattern indicates that the original data y versus x has an exponential pattern. Include scatter plots and graphs of models corresponding to the three data sets contained in this lab. Determine the annual percentage change for all your models. Explain how modifying your models by a constant multiple affects those percentage rates. Discuss your predictions for the turtle's extinction, taking care to explain your interpretation of "extinct for all practical purposes," and your prediction for upgrading the turtles' status to "threatened."

Even though students often complain about the work involved in writing lab reports, they frequently cite these activities as valuable learning experiences in their course evaluations. Two sample comments follow:

- Writing lab reports . . . helped me not only learn appropriate and correct mathematical language, it forced me to confront areas of understanding about which I was unsure. Writing about larger concepts and how math is applied made me clarify connections on an entirely different level.

- I liked the AIDS, SAD, and Turtles labs because they put math in the context of situations that were not mathematical. . . . These labs forced me to understand all the information (graphs, equations, derived functions, etc.), but then it was another, separate process to tie in each bit of info . . . to explain its relevance.

Sample Exploration: *Copycats*

The *Copycats* exploration guides students through an investigation into how the parameters A, B, C, and D affect the graphs of $f(x) = A\sin[B(x-C)] + D$ and $f(x) = A\cos[B(x-C)] + D$. In particular, students often equate B and the period. Questions 3 and 4 challenge this misconception. Below are parts (a – c) of Question 3:

The period of a sine or cosine function means the width of the smallest interval that contains one complete fundamental wave. What is the period of sin(x)? Of cos(x)?

(a) Speed up the sine wave: Fit two sine waves into the interval [0, 2π]. What formula did you use to create the graph?

(b) Now fit three waves, four waves, five waves into that interval. Write the formula for each function you used.

(c) How would you slow the wave down, causing it to use twice as much horizontal space to complete its fundamental pattern? Write the function.

By the end of question 4, students discover the inverse relationship between B and the period, namely that the period equals $2\pi/B$. Because students discover this formula, they tend to remember it and have an easier time applying it.

Sample Project: *Fuels Rush In*

The project *Fuels Rush In* centers on two-years' (minus one month) worth of data on the amount of gas used in one of the authors' homes (therms) and the average monthly temperature (degrees Fahrenheit). The monthly data, starting with January 1994 (month 1), appear in Figure 5. Unfortunately, the author was missing the bill for March 1995.

	1	2	3	4	5	6	7	8
Therms	81	75	57	37	17	13	11	10
Temp.	18	15	28	41	54	61	62	65

	9	10	11	12	13	14	15	16
Therms	11	17	22	48	72	68	?	40
Temp	62	55	52	36	32	23	?	40

	17	18	19	20	21	22	23	24
Therms	30	14	10	10	11	12	27	54
Temp.	53	60	63	65	62	59	46	29

Figure 5. Data on amount of gas used and average monthly temperature.

First, students plot gas usage versus time and temperature versus time on the same set of axes. They describe what is similar about the two plots and what is different. In addition, they answer the following question: What, if anything, do these plots tell you about the relationship between temperature and gas consumption? Calculator-created plots appear in Figure 6.

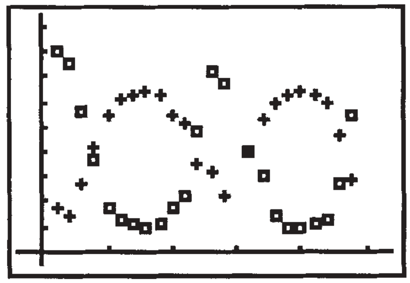

Figure 6. Gas usage versus time (boxes)
and temperature versus time (crosses).

Next, students apply what they learned from the text and/or the *Copycats* exploration to fit sinusoidal models to gas usage versus time and temperature versus time. They compare their models to models fit using their calculator's regression capabilities. Finally, they use their models to predict the gas usage and temperature for the missing month, March 1995.

Incorporating labs, explorations, and projects (such as the examples contained in this paper) in a precalculus course, make Precalculus a much richer course than its purely theoretical counterpart. In this environment students are more motivated to study mathematics. Instead of viewing mathematics as separate from the real world, they are convinced that mathematics is integral to interpreting real-world phenomena and solving real-world problems.

Reference

[1] Moran, J., Davis, M., and Murphy, M. *Precalculus: Concepts in Context 2e* Brooks/Cole Publishing Company, 2004.