VISUAL ALGEBRA WITH TECHNOLOGY

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Abstract

Whether reform or constructivist minded in pedagogical preference, whether traditional or modern in outlook, there is no denial that our teaching methods need visual components to succeed. This paper is about bringing the visual world into the mathematics classroom in such a way that its precepts and ideas can be fully applied to our visually ready students. As important as it is for students to learn via these constructs, it is equally important that our mathematics instructors are both cognizant of and trained to focus visual technology upon the processes of teaching and learning.

Including a technology component in our mathematics classrooms is now strongly regarded as essential, if only to gain the interest of the students in what we are trying to do. It is important to create visual devices that convince students that mathematics has importance, value, and can be understood. What we are suggesting is using visual language to transport our students to a deeper world of mathematical relationships, variables and ultimately to problem formulation and solution.

Introduction

Two distinct populations comprise the world of College Algebra. In a 1990 study [Albers, Loftsgaarden, Rung, & Watkins, 1992] of 1,295,000 students studying mathematics in community colleges, 56% were studying at the remedial level while only 19% were studying precalculus. In four year colleges and university, the figures were somewhat reversed with 34% studying precalculus and 15% studying remedial mathematics. In the fall of 2000, more than three million students were enrolled in mathematics courses at two and four year colleges, of which 22% were enrolled in precalculus classes. Furthermore, what is generally unrealized to many mathematics instructors is that the vast majority of students enrolled in College Algebra courses have no plans (or are required) to take any more college level mathematics courses. Indeed, while enrollment in Calculus I has remained constant over the past two decades the enrollment in college mathematics courses has increased 18%. The bulk of this difference is accounted for in non-calculus based mathematics courses such as elementary statistics (+65%), mathematics content course(s) for elementary teachers (+55%), and Liberal arts mathematics course (+33%) [McGowen, 2002]. Nonetheless students enroll in College Algebra courses in far greater numbers than all other precalculus courses combined. Thus, College Algebra courses serve two populations, those going on to higher math and those ceasing their mathematics with their current course. The challenge to make an algebra course interesting, important, and at full strength for both groups, is formidable.

Computers and calculators can be used to great advantage to accomplish this program. We propose that they can be used to approach the gamut of goals including (1) constructing knowledge, (2) validating knowledge, (3) discovering knowledge, and (4) similating knowledge - all from a visual viewpoint. Indeed, at one time or another we have all tried to deliver material from at least one of these tacks. This leads us to the concept of *visual knowledge*. Many of our students are extremely visually oriented, and though we may have some idea of what this means, the concept has not really been defined. Before moving to a definition, it should be of some value to indicate how visual information and knowledge have insinuated themselves into our thinking. Internet search engines, seemingly the source of all things, generates many hits to the input "visual knowledge." Remarkably, there is an entire new field called *visual knowledge engineering* as roughly a sub branch of knowledge engineering [Eisenstadt, 1990]. It is designed to traverse and visually organize an emerging knowledge store and the semantic space of the domain in the most natural form. As an example, there is an "image panel" or a sketchpad for the concept maps, diagrams and pictures. What is remarkable is that even in library science, for example, we see the use of visual knowledge engineering to clarify the semantic space of courses using mathematical techniques such as singular value decompositions [Börner, 2000].

Concept maps are particularly important ways of categorizing and storing information in compact form. Easily remembered, the concept map forms an important tool in the way we teach the shapes of graphs and properties of functions. A companion to concept maps are process maps, which through variations of flow give a visual picture of how to solve mathematical problems.

With this in mind we define *visual knowledge* at the most primitive end to be a visual awareness of particular image structures. This can be measured by way alternate image structures can be discriminated to a verbal or even analytical description of the awareness. At the higher end, we define visual knowledge to be an understanding of variations and dependencies as evident from visual representations of the information. The range is broad in correspondence to the meaning of general knowledge, reflecting relative depths of visual understanding and knowledge. It is important to include that the imagery can be dynamic as well as static. As well it includes that information contain by data may have a fully meaningful visual knowledge dimension. Visual knowledge is the companion to both analytical knowledge and data knowledge.

In the context at hand, we propose a modified version of College Algebra to be called *visual algebra*. In rough terms, this means the application of visual information and cues to generating and accumulating an understanding, meaning, and belief in algebra. A special emphasis is placed on identification of the right function to use in a given situation. While not new in totality, it is new in its comprehensiveness. Its implementation is pervasive, from devices to be employed by the instructor to artifacts to be used by the students, to interactive tutorials on the Web. If the visual part meant just more pictures, we would be suggesting little more than extant methods. If the visual part meant just data application, we would suggest little more than a portion of certain reform algebras. If the visual part meant just interactive exploratories, we would be suggesting just a collection of enchanting artifacts. What we intend is all of these and far more. While not pedagogy *per se*, visual algebra is a collection of teaching methods designed to capture the imagination of the student.

The components proposed here, if anything, increase the responsibility of the teacher, or perhaps better named the instructional producer. The teacher here must select from a myriad of visual possibilities and select those that are just right for their class at the given moment. For example, one might employ a visual relevant to an important news event such as the opening of a new suspension bridge or a new discovery about the atom. Though less appealing to the scientist, information about the movie box office receipts provides an interesting entrée to several types of functions.

Changing the curriculum is a common thread throughout mathematics these days. Many works have been devoted to the reshaping of what we teach. There seems to be a feeling that by just making the proper changes to the curriculum, it will be possible to make the connection to the student. Calculus, after the emergence of tools like Mathematica and Maple, has been extensively hit. In a way, Calculus will never again be the same (Allen, 2001, 2003). Some attempts at making mathematics more of a lab science have also been offered. For example, in the book, Learning by Discovery: A Lab Manual for Calculus, we obtain a collection of laboratories that can make learning more interesting and relevant (Solow, 1993). The recent book by Devlin (1994) anticipates a visual framework, though many patterns are syntactical, numerical, and symbolic.

The visual world

Having achieved some level of visual literacy, the student may be said to have some degree of "visual intelligence" – being visually smart, which is more of a calibrated for of visual literacy. In the context of mathematics, this means the ability to determine mathematical patterns and relations. A transformational ability, the student must convert visual information into qualitative mathematical information and ultimately to quantitative information. Finally, we arrive at the student's "visual IQ," which measures one's level of analytic knowledge that is queued from visual information.

Visual information transformed

Our primary goal is to use visual information to engage the student in active learning by stimulating their belief system to the conclusion that algebra has value. The instructor, the main player in this process, must be a part of the plan from the onset. To say that this is student-centered learning is not accurate, but to say that the entire script is written to attract, guide, and interact with the student to learn is. Our plan for visual intervention in the learning process is expressed in a visual manner in the following figure.

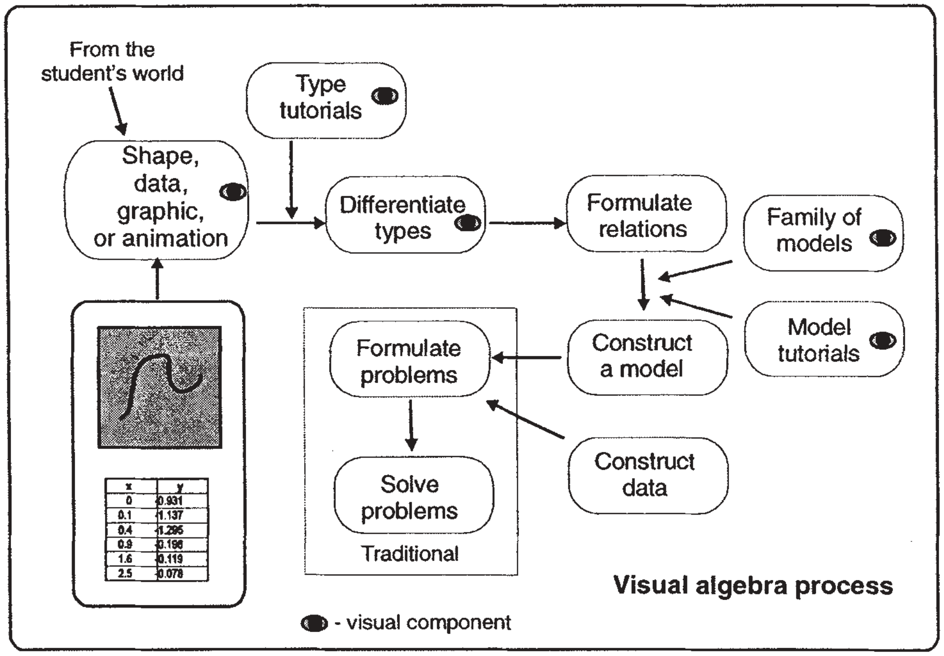

Figure 1 – The visual process in algebra

Visuals and Visual Tools

Many visual tools have been used over the centuries; the most important being static graphics, images, or photos. The best texts have always had just the "right" picture accompanying the right example. It was interesting and it led to interesting mathematics and solid learning. Indeed, long after the course the image often provided a memory link to certain mathematical situations in other contexts. As well the composition of the picture was important.

A variety of tools can be successfully applied to teaching algebra. A partial list of visual tools for the students includes spreadsheets, animations and animation editors, interactive applets, animations, and images, digitization, CBL/MBL, and symbolic algebra software. The corresponding list for the developer and instructor includes all of these together with screen capture software, video capture (and camcorders), and images, and photo/graphics editors. Spreadsheets have enormous potential in the classroom, are continually being re-discovered. Not only are they available ubiquitously, they are easy to use and are remarkably versatile.

Other important visual factors and visual benefits are contained in the following points.

- Using motion such as an animation is one of the more powerful enhancements to almost any teaching function is motion, of which one kind is animation. The power of animations is that they show or point to what is to be observed.
- The answer to the question "What can an online course do that a classroom teacher cannot do?" is partly answered through animations. Cleverly created

animations can reveal more about functional and geometric relations than almost any other teaching tool.

- Digitization is an old technique that high-energy physicists use to use to determine tracks of particles in a bubble chamber. The idea is to take an image, impose a coordinate system, and with respect to the coordinate system, coordinatize (i.e. digitize) selected point. Then use the data to model a function.
- Software computer algebra systems are so powerful that they essentially do all of the algebra, leaving little for the student. Hence cleverly designed projects and applications must be created to allow students to learn and want to learn in the face of this enormous computing power. Computer algebra systems, can be used however by instructors to greatly enrich the learning environment.

The parabola/quadratic is a conic and more

What is a parabola? The answer can be given in several layers of depth. In the list below it is possible to see an evolution of understanding of the parabola as a layered process, with each layer adding more and deeper understanding. Note how many are visual.

1. A curved shape, as in a convex (or concave) shape with symmetry. (Shape)
2. A conic. That is a member of a family of curves and distinguished from others in the family. (Classified shape on the basis slicing a cone.)
3. Area of a rectangle with variable sides in fixed arithmetic or geometric proportion. (Application)
4. A relation between focus and directrix. (Proportion between distances)
5. Relationships based on the square law for proportionality. (Algebraic/geometric)
6. A relationship with one free variable (*a*) to determine aperture:
   $y = ax^2$ (Algebraic)
7. A relationship with three free variables (*a, h, k*) to determine aperture, and vertex
   $y - k = a(x-h)^2$. (Algebraic)
8. A polynomial curve. A nonlinear curve. (Class based on more general algebraic relationships)
9. The solution of a differential or difference equation. Motion based on uniform acceleration. (Modeling)
10. The path of a trajectory such as a bouncing ball, basketball, cannon ball. (Application)
11. The shape to reflect incoming rays to a central point such as such as radio and satellite receiving dishes, solar collectors, reflecting telescopes, distance microphones. The shape taken by the free surface of the liquid contained in a rotating cylinder is a paraboloid. (Application)
12. Velocity of a falling body. (Application)

Similar categories and lists can be constructed for other functions – all with significant visual components.

Summary

Clearly the time of visualization is upon us. The students have already arrived and are waiting for their instructors to come on board.

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