SLIDES, TUBES, WEDGES, AND RAMPS ON DERIVE5

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Introduction

DERIVE is a user-friendly piece of software that students are able to use independently in a relatively short amount of time. In this article I will discuss how to use DERIVE to help students visualize in 3D.

Slides

To visualize a space curve using DERIVE one generally Authors a vector-valued function and then plots it in a 3D window. For instance, one might Author the vector $\left[4\cos(2\pi t),\quad 4\sin(2\pi t),\quad 5t\right]$ and then plot it to get the helix shown below.

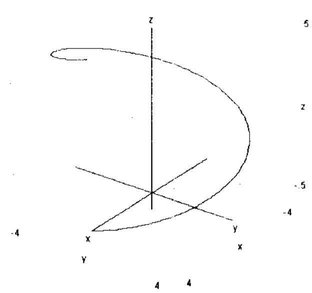


Figure 1: Standard Helical Plot

Alternatively, one could visualize the helix as the top of a slide. To do this, one would first Author and then Simplify and plot the expression below. (See Figure 2)

$$\text{VECTOR}\left(\left[4\cos(2\pi t), 4\sin(2\pi t), 0; 4\cos(2\pi t), 4\sin(2\pi t), 5t\right], t, 0, 1, 0.025\right)$$

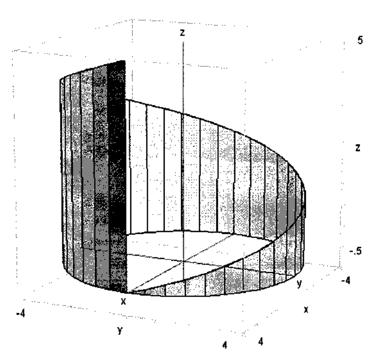


Figure 2: Helix Viewed as a "Slide"

We will now consider another way to use "slides" to visualize in 3D. Suppose we wish to find the path of steepest descent on the surface $z = f(x,y) = 100\left(1-e^{-x^2-\frac{y^2}{2}}\right)$. To do this we must always proceed in the direction of $-\nabla f$ in the xy-plane. By setting the slope, $\frac{dy}{dx}$, equal to $-\nabla f$ and solving we can find the curve in 2D whose tangent vector always points in the direction one would have to proceed to descend most rapidly. When this path in 2D is projected up to the surface, it gives us the path of steepest descent. I have graphed both the 2D curve and the path of steepest descent in the figure below. To get the graph on the right, I Authored, Simplified, and plotted the following expression.

$$\text{VECTOR}\left(\left[t,\ -2\sqrt{t},\ 0;\ t,\ -2\sqrt{t},\ 100\left(1-\hat{e}^{-t^2-2t}\right)\right],\ t,\ 0.1,\ 1,\ 0.05\right)$$

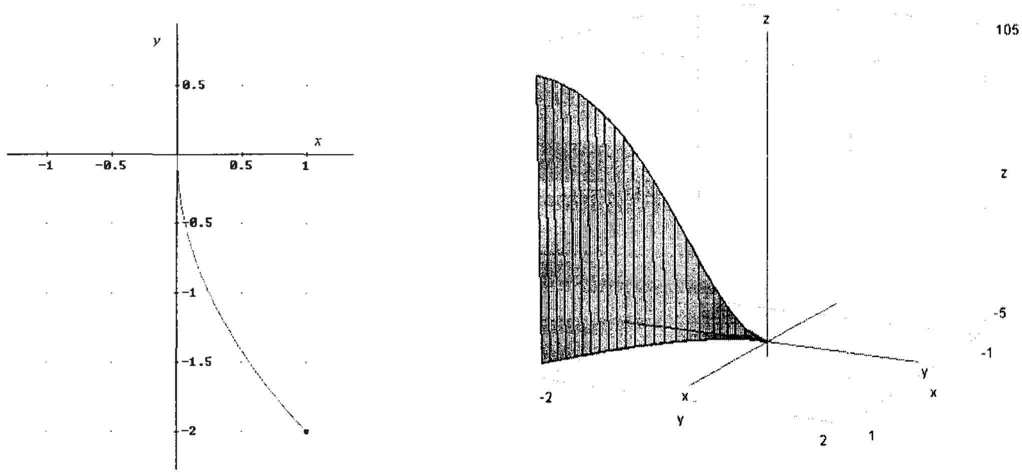


Figure 3: Path of Steepest Descent Viewed as a "slide"

When I've asked my students whether the particle is rising or falling as you traverse the path in the 2D plane from $(1,-2)$ to $(0,0)$, I've had students look at the graph on the left above and tell me that the particle is rising (because y is increasing), but I have never had a student look at the graph on the right above and tell me that the particle is rising. The 3D view makes it painfully obvious that the particle is falling – this is good!

<u>Tubes</u>

Sometimes students are asked to show that a curves lies on a surface. They are expected to do this analytically, and rightly so, but it does help if they can actually see what they are doing. For instance, suppose I ask my students to show that the helix

$\langle 3\cos(t),\ \ 3\sin(t),\ \ t\rangle$ lies on the circular cylinder $x^2 + y^2 = 9$. Before I take them through the paces of how to show this, I would like to show them a picture. Plotting the cylinder in 3D and then plotting the helix doesn't work well because the helix gets "lost" on the cylinder, i.e. you can't see it very well because the helix is so thin and the cylinder is so colorful. You get a much better picture if you plot a small tube whose centerline is the helix and plot the cylinder as a wire frame diagram. To plot a "tube" in 3D, we use the TNB reference frame. The commands to get DERIVE to compute the Tangent, Normal, and Binormal vectors are given below.

```
r2(t) := [3·COS(t), 3·SIN(t), t]
```

$$T2(t) := \frac{\frac{d}{dt}\,r2(t)}{\left|\frac{d}{dt}\,r2(t)\right|}$$

$$\left[-\frac{3\cdot\sqrt{10}\cdot SIN(t)}{10},\ \frac{3\cdot\sqrt{10}\cdot COS(t)}{10},\ \frac{\sqrt{10}}{10}\right]$$

$$N2(t) := \frac{\frac{d}{dt}\,T2(t)}{\left|\frac{d}{dt}\,T2(t)\right|}$$

$$[-\ COS(t),\ -\ SIN(t),\ 0]$$

```
B2(t) := CROSS(T2(t), N2(t))
```

$$\left[\frac{\sqrt{10}\cdot SIN(t)}{10},\ -\ \frac{\sqrt{10}\cdot COS(t)}{10},\ \frac{3\cdot\sqrt{10}}{10}\right]$$

The tube is plotted parametrically by Authoring, Simplifying, and plotting the following expression.

```
tube2(s, t) := r2(t) + 0.05·COS(s)·N2(t) + 0.05·SIN(s)·B2(t)
```

In the above expression, **r2**(t) is the position vector of the helix and the radius of the tube is 0.05 units. Plotting the tube and a wire frame diagram of the circular cylinder yields the below figure in which it is clearly seen that the helix lives on the cylinder.

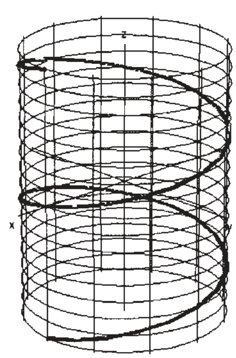


Figure 4: A Helix on a Cylinder

Wedges

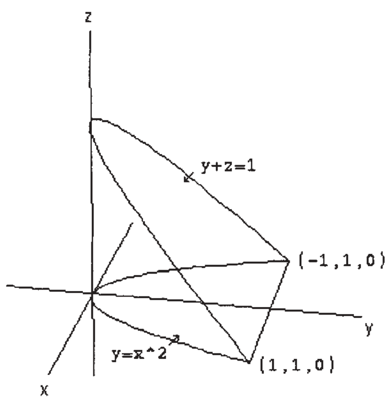
Suppose we wish to find the volume of the solid region enclosed by the plane, $y+z=1$, the parabolic cylinder, $y=x^2$, and the coordinate plane, $z=0$. The solid region is a parabolic wedge and can be represented using a wire frame diagram, which is given below and to the left, or as an enclosed region, which is given below and to the right. The wire frame diagram it more useful to help students get the limits of integration correct, whereas the other diagram helps students see what the solid region actually looks like. To get the graph on the right below, you Author

$$\text{VECTOR}\left(\left[x,\ x^2,\ 0;\ x, x^2,\ 1-x^2\right],\ x,\ -1,\ 1,\ 0.1\right)$$

followed by Authoring

$$\text{VECTOR}\left(\left[x,\ 1,\ 0;\ x, x^2,\ 1-x^2\right],\ x,\ -1,\ 1,\ 0.1\right)$$

and Simplifying and plotting each expression in turn. Although the math is the same with or without these diagrams, the students are more comfortable working through a problem when they have a picture of it. Furthermore, they are less likely to make silly mistakes on the limits of integration if they have a diagram to guide them.



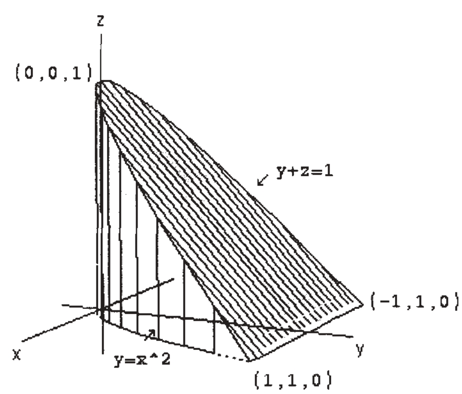


Figure 5: Parabolic Wedges

Ramps

Wire frame diagrams are useful in helping students find the correct limits of integration for other solid regions as well. For instance, to give my students practice interchanging the order of integration I might ask them to find the volume of the solid region enclosed by the surfaces $x=0$, $x=1$, $z=0$, $y=-1$, and $z=y^2$, using different orders of integration. Giving them multiple copies of the below diagram to mark up as they draw in the differential area elements has proven to be a helpful exercise for them.

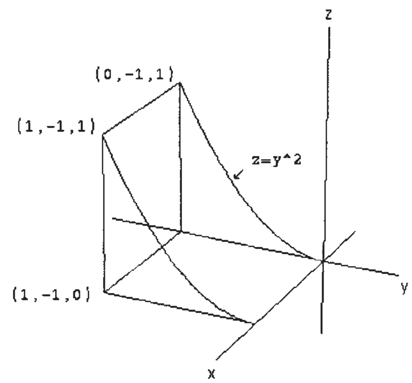


Figure 6: Wire Frame Ramp

To obtain the above wire frame diagram one would Author and plot the below expressions in a 3D window one at a time. The range of all three axes is –0.5 to 1.5.

[0, -1, 0; 0, -1, 1]
[0, -1, 0; 1, -1, 0]
[1, -1, 1; 1, -1, 0]
[1, -1, 1; 0, -1, 1]
[1, -1, 0; 1, 0, 0]
[1, y, y^2] on $-1 \le y \le 0$
[0, y, y^2] on $-1 \le y \le 0$

Conclusion

DERIVE is a useful tool in multivariate calculus because it helps the students to visualize the symbolic expressions they are working with. It is also a versatile tool for plotting many different kinds of three-dimensional objects such as slides, tubes, wedges, and ramps. All of these objects help the students to visualize what they are doing and hence help them to learn the mathematics.