TECHNOLOGY-SUPPORTED INQUIRY-BASED LEARNING IN COLLEGIATE MATHEMATICS

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Abstract

The paper discusses how Mathematica (a Computer Algebra System) activities are implemented in a matrix algebra course to provide and support inquiry-based learning both during and out of the class. Students' comments, instructor's experiences and course material are used to discuss pedagogical issues that have occurred while integrating technology.

Introduction

According to constructivist view of learning, learners construct their own understanding of subject matter. To achieve the goal, learners will need learning environments supporting investigation, conjecture and discovery. Inquiry-based approach is one that provides and supports learning environments where learners observe events, ask questions, construct explanations, test those explanations, use critical and logical thinking, generalize observed patterns, and consider alternative explanations. Inquiry-based learning environments can be structured in various forms. In one, learners are provided guided, hands on activities and are required to arrive at their own conclusions through experimentation, observation, investigation and conjecture. One may also provide environments where learners are required to design and carry out experiments. The former is the learning environment provided in the matrix algebra classes discussed in this paper.

Research [6, 8] reports that students learn much more from an organized lecture when they are given inquiry based learning environments to first experiment on specific information relevant to a topic. One way we can provide inquiry based learning environments for our students is through interactive interfaces which guide students through a process of inquiry learning [9]. Wicks adds "…*Mathematica* and Maple are two such systems with which we can create rich learning experience for our students." There has been a wide range of computer activities [7] such as those of the ATLAST project [5] and Wicks interactive approach [10] used in teaching first year linear (matrix) algebra concepts. With the few exception, most CAS activities do not provide learning environments to maximize higher-level cognitive activities such as investigation, conjecture and generalization. In order to advance students' understanding of abstract linear algebra concepts, the author worked on classroom activities supported by *Mathematica* to provide inquiry-based learning environments. The rest of the paper details three attempts to implement *Mathematica* interactive activities as well as pedagogical issues that occurred. A sample *Mathematica* activity is also described to discuss how technology-based activities can be designed to maximize higher-level cognitive activities.

Linearly Independent (Dependent) Sets

Example 2

- **Run the commands given in the following *Mathematica* cell.**
- **Based on the outcome of the commands, answer the following questions (write your responses in a new cell right after the cells with *Mathematica* Commands and Output):**

    1. State the solution for the vector equation a **i** + b **l** + c **j** = **0** where **i** is the vector in green, **l** is the vector in blue and **j** is the vector in red.
    2. Is the set {**i, l, j**} linearly independent? Explain your reasoning.

- **Enter your own vectors from R^3 in to the cell that comes right before the cell with the *Mathematica* commands, and name them as i, l, j and k, and next run the cell.**
- **Repeat the steps 1 and 2.**
- **Now, for the set of vectors you have used in step 3, discuss whether the set { i, l, j, k} is linearly independent or not.**
- **Discuss Span of the same set {i, l, j, k}.**

In[1]:=

```
picture = BasisPicture[{l, j}, 2, HeadScale → .0003, TailWidth → .0001];
g = Show[picture, Axes → None];
u1 = Vector[k, Color → Hue[1], TailWidth → .01, HeadScale → .05];
u2 = Vector[j, TailWidth → .01, Color → Hue[1], HeadScale → .3];
u3 = Vector[ l, Color → Hue[.5], TailWidth → .01, HeadScale → .3];
u4 = Vector[ i, Color → Hue[.7], TailWidth → .01, HeadScale → .3];
Show[g, u4, u3, u2, Axes → True, AspectRatio → Automatic, ViewPoint -> {0.293, -2.185, -
```

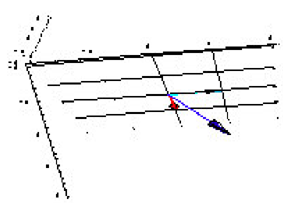

Figure 1. *Mathematica* notebook addressing linearly independent (dependent) vectors, span and spanning set. This shows how the notebook looks after running Mathematica commands. Tittles are written in red, and instructions are written in blue. Modified from Wicks [10].

Sample Mathematica Notebook

Mathematica notebooks containing *Mathematica* commands, some of which are modified from the *Mathematica* Packages by Wicks [10], are written by the investigator as interactive, guided supplements to lectures. They are composed of interactive cells of examples and non-examples of basic linear algebra concepts. Emphases are given to two- and three-dimensional *visual* demonstrations of basic vector space concepts. For instance, *Mathematica* notebooks similar to the one in figure 1, through supporting mental images and structures [3], are used to help students gain deeper understanding of the formal (abstract) definition [4] of linear independence, dependence, span and spanning sets.

Stages of Implementation

First attempt: First implementation was done in the fall 1999 semester [1, 2]. *Mathematica* activities similar to the one in figure 1 were used to support in-class experimentation on various vector space concepts such as linear independence and span. The *Mathematica* activities used in this implementation were also tested with a group of linear algebra students during the 1999 summer session.

- Class met in a computer laboratory where each student had access to a computer.
- Students were given a short tutorial on basic *Mathematica* functions such as opening files, editing and running *Mathematica* cells.
- Relevant *Mathematica* notebooks were provided to students before the start of each class meeting.
- During class, after the initial introduction and discussion of *Mathematica* examples corresponding to examples stated on blackboard, students were asked to experiment on their own examples relevant to the subject matter.
- Students were given 15 to 20 minutes (at times, it took longer than 20 minutes) for each activity. At the end of their experimentation, they were to discuss and test their observations, findings and conjectures.
- Lessons were extended based on students' findings and conjectures they arrived at as a result of in-class experimentations.

The pedagogical issues that occurred while integrating *Mathematica* activities to support in-class experimentations were:

- **Time**: Since class time was allocated for in-class experimentations, detailed coverage of all the topics included in the syllabus was not possible. The instructor had to pick and choose to insure detailed coverage of basic and necessary concepts. The other concepts were briefly introduced in lessons and assignments.
- **Infrastructure:** Even though each student had access to a computer in class, not all worked properly.
    1. At times, the instructor had to interrupt class to deal with malfunctioning computers. Due to lack of memory, computers were not able to handle *Mathematica* graphical outcomes.
    2. **Classroom setting:** Some students spent class time on the Internet. Since the laboratory had desktop computers, it was difficult for the instructor to maintain control of students' activities.
- ***Mathematica:*** Providing *Mathematica* commands with the activities resulted in some students shifting their attention from mathematics to the commands.

Second attempt: In light of the findings from the first study, the author attempted to implement the same set of *Mathematica* notebook activities with minor modifications into a matrix algebra class at a different university during the spring 2002 semester.

- Students were again given a short tutorial on *Mathematica*.
- This time, *Mathematica* activities were included in worksheets assigned as homework. Students were to use corresponding *Mathematica* activities to answer guided questions provided on worksheets. Then through experiments they were to arrive at, discuss, and test conjectures. Furthermore, during the following class

meeting, students were to discuss their responses to the questions provided on worksheets as well as their findings from experiments.

The pedagogical issues that occurred during this implementation were slightly different.

- **Cognitive Skills**: For a majority of students this was their first time using worksheets and working without instant guidance from the instructor. This made it necessary for them to collaborate with others in class while working on worksheets. Most lacked the skills necessary for effective collaboration, thus they required more guidance than the worksheets alone provided.
- **Lack of *Mathematica* Knowledge:** Students showed difficulty dealing with *Mathematica* related errors. Since they did not have substantial knowledge of *Mathematica*, they needed help often enough that it became a major learning obstacle.
- ***Mathematica* notebooks** provided with the worksheets required the presence of *Mathematica* on computers. The majority of students had to come to campus to complete worksheet assignments. A considerable number of students had full time jobs (29%), making it difficult for them to complete their assignments on time.

Because of the significant negative effect the last two issues had on students' performance, the instructor had to switch to in-class demonstrations later in the semester. Students were shown demonstrations using the same *Mathematica* activities, and asked to discuss their observations. The following semester the same instructor used the *Mathematica* activities strictly for in-class demonstrations. After each demonstration students were to discuss *Mathematica* outcomes first in groups of three to four and next as a class. Discussions on demonstrations took about 10 minutes. The pedagogical issues that occurred during in-class demonstration of *Mathematica* activities were:

- **Inquiry** was at a minimal level (or no inquiry occurred) due to time constraints.
- **Time**: Since class time was allocated for in-class demonstrations and discussions, detailed coverage of all the topics that were included in the syllabus was not possible. The instructor had to pick and choose to ensure detailed coverage of basic and necessary concepts. The rest was briefly introduced in lessons and assignments.

Current attempt: Currently the author has been implementing an online laboratory approach in a matrix algebra section. The online activities [11] are developed based on the same set of *Mathematica* activities that have been used in the earlier attempts. So far, students have worked on three online activities. The feedback received from the students enrolled in this particular course indicates that the implementation seems to have proceeded with no major difficulties. The only problem students encountered was not with the laboratory activities but with WEBCT, a course site where students are asked to post their reflections discussing their experiments and conjectures. Some of the observed benefits of the online implementation are:

- Covering the course material does not seem likely to become an issue. So far, the instructor has been able to cover the scheduled course material in time.
- Minor problems with the activities have been resolved without breaking the flow of the lessons and students' progress in and out of the class. For instance, when the instructor realized a significant number of students did not know how to enter vectors into an online activity [11], s/he did not have to wait until the next class meeting to

inform students. S/he was able to include a short explanation on the activity the minute the problem was realized.

- **Inquiry**: This approach provides almost all the aspects needed for effective inquiry based learning environments. Students have plenty of time to experiment on the activities at their convenience. Students come to class better prepared for discussions on relevant topics, which seems to help students better understand the material covered in class [6].
- Another aspect of the online activities is, thanks to Web*Mathematica* provided by Wolfram Inc. [12], that these activities allow real-time graphic manipulation. This option was not possible on the activities used in the earlier implementations.
- The online approach does not require the presence of *Mathematica* on computers, which allows students to use any available computer with Internet access.
- This approach has fewer factors that distract students. Thus students have more time to focus on mathematics as opposed to technology.

Conclusion

The paper discussed three attempts to integrate *Mathematica* activities to provide and support inquiry-based learning environments. Most of the data reported is the result of ongoing research investigating the effect of technology on teaching and learning mathematics. The author will continue the investigation, and further discussion of the results will be the topic of a future paper. The three attempts have provided evidence that many factors influence effective implementation of technology in providing and supporting inquiry-based learning environments. The online laboratory approach seems to be the most effective implementation that can provide and support inquiry based learning, especially for courses at schools where many students hold full time jobs.

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