QUANTITATIVE MATH MODELING AND EXPERIMENTS

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Introduction

Visual images provide readily accessible quantitative data to illustrate, support, and verify quantitative models from a broad range of fields. Roughly speaking

"If you can see the data. You can get the data".

I will demonstrate a simple Java plugin (developed by the author with the assistance of Robert Pastel and Alok Mishra, Computer Science, MTU) that extracts data from curves in images. The software inherits an extensive suite of pre-processing algorithms from the freeware image viewer ImageJ (maintained by the National Institutes of Health) that it extends. The plugin saves numerical data describing the curve in the image in a straightforward format that can be readily imported, analyzed, and graphed with any spreadsheet or computer algebra system.

The balance of the paper:

- Demonstrates an application of this software package to an oscilloscope image.
- Examines the shape of suspensions bridges, suspended cables, and the Gateway Arch using the package.

The experiments and software described in the paper are part of an ongoing effort at Michigan Technological University to develop simple and effective experiments tailored to support the applications material commonly presented in ODE and Calculus courses. This effort was prompted by a student survey that showed that students are extremely skeptical concerning the utility of the applications (and the expertise of their math instructor to present them) presented in their Math courses.

Java Plugin

Figure 1 is a frame extracted from a video of an oscilloscope screen. It is actually the amplified and trace of the voltage generated by a pair of photo-diodes as a brightly lit, microscopic particle flies by. The question of interest is what "*shape*" are the pulses. Are they Sech "a + b Sech[c (x-x0)], Gaussian "a + b Exp[-c (x-x0)^2]", Lorentzian "a + b/(c + (x-x0)^2)", or some other shape.

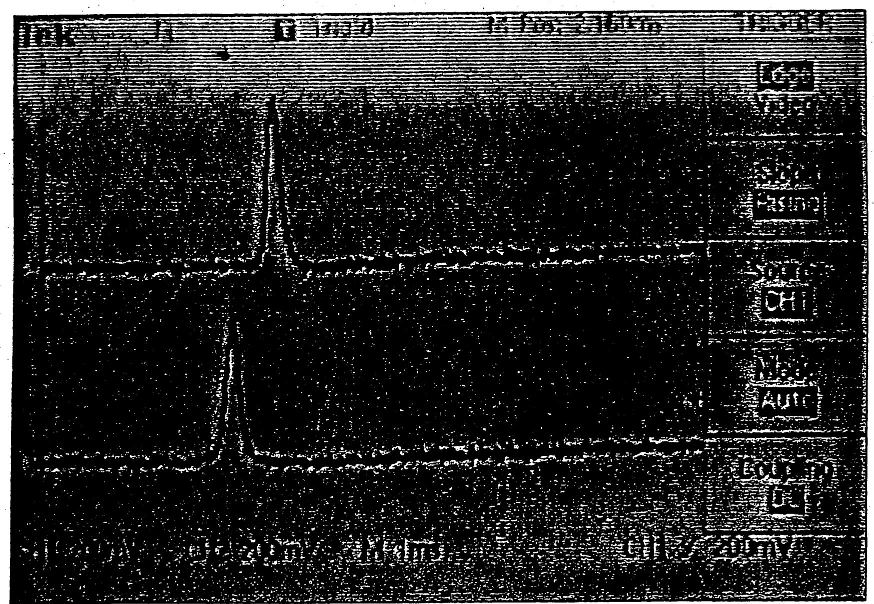

Figure 1: Frame from an Oscilloscope Movie

The first task is to extract the data from the image using the Java plugin shown in the screen-shot in Figure 2.

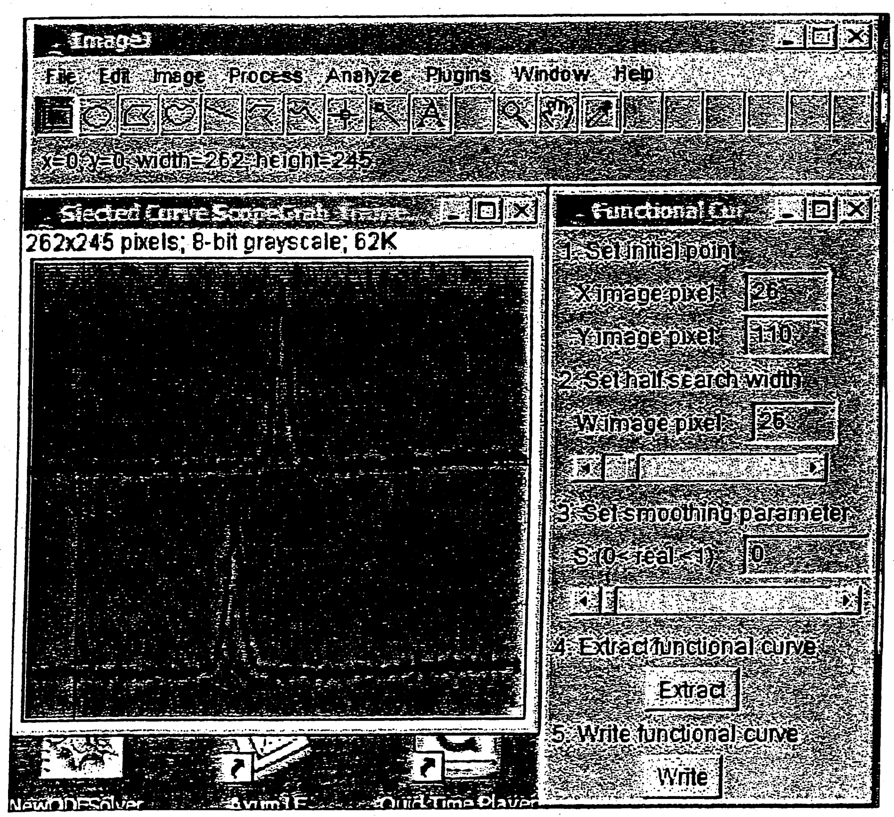

Figure 2: Data Extraction Plugin ScreenShot

The window on the right controls the data extraction while the top window contains the underlying JImage controls. The extracted curve is shown in a contrasting color overlaid on the image: unfortunately the bright red selected for the contrasting color does not show well in gray scale.

The data extraction follows a simple, robust, and easily explained algorithm. When the "Write" button is pushed a display of the numerical data in units of pixels is displayed and a prompt appears to save the data to a file in the standard spreadsheet interchange format. The saved data can then be imported into your favorite computer algebra system and fit to the proposed functional forms to determine the best "shape".

The result using the *Mathematica* **NonlinearFit** command are the three potential fits shown in Figure 3. It is pretty clear that the Gaussian fit is best. Before we did the fit all the proposed curves "look" pretty similar but one clearly provides a better explanation of the data than the others.

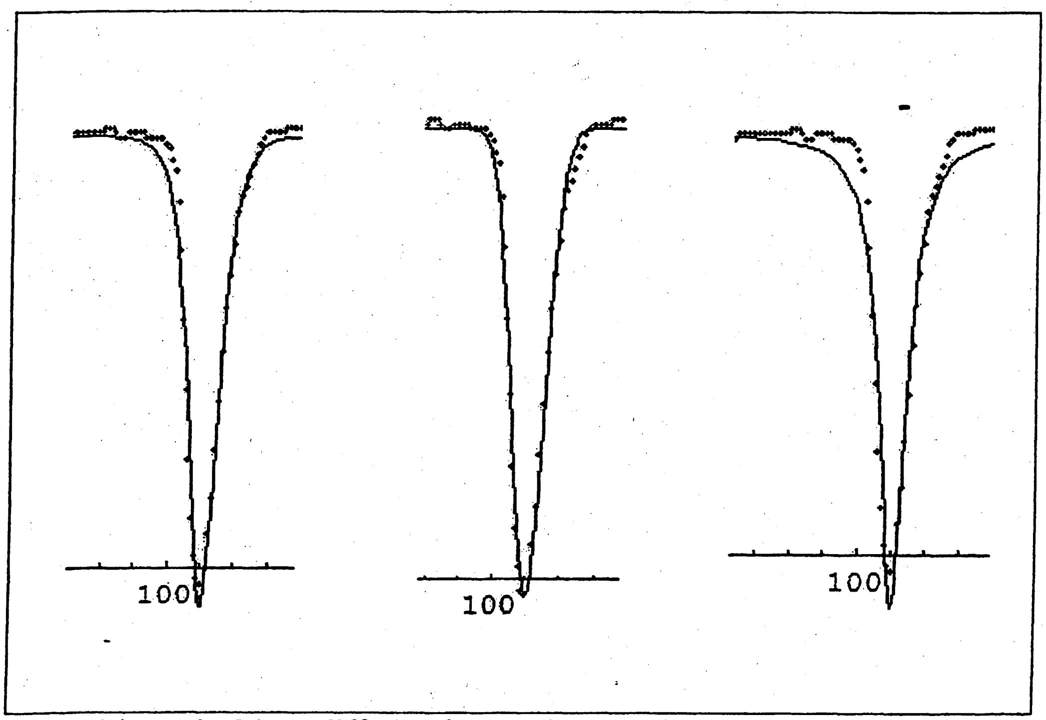

Figure 3: Three different fits (Sech, Gaussian, and Lorentizian)

Of course the *Mathematica* "fit" command provides detailed values (and confidence intervals) for the parameters in the Gaussian fit, which can be compared to the expected values.

Cables, Bridges and The Gateway Arch

Undergraduate students rarely see quantitative comparisons. All they are ever shown in beginning math and science classes are qualitative comparisons. Without a detailed fit the three models in the previous section are identical! On the same level of detail a portion of a parabola is indistinguishable from a portion of a catenary. We have the technology to demonstrate that a hanging cable forms a catenary rather than a parabola and that the support cables of most suspension bridges form a parabola. This reinforces the assumptions underlying the different models, helps demonstrate that the models are capable of providing excellent agreement with the real world, and provides credibility for other more complicated models we present. Figure 4 contains images of a suspension bridge and a suspended cable above a bridge.

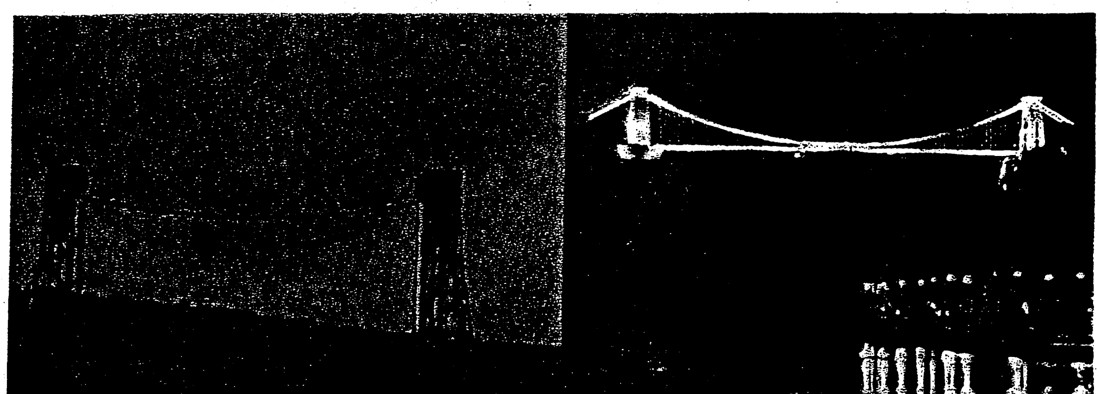

Figure 4: Hanging Cable and Suspension Bridge

Extracting the data using the plugin allows one to directly compare the shapes of the cables in the two pictures. The conclusion is inescapable the hanging cable (which supports only its own weight) in Figure 4a is a catenary while the brightly lit top cable in Figure 4b (which supports the heavy, horizontal roadbed) is a parabola.

The simplest way to do this is extremely natural in an ODE or calculus class: rather than comparing the data to the analytical solution of the ODE one compares the data directly to the ODE derived from the vertical force balance in the cable.

An interesting question is what shape is the Gateway Arch? Is it a catenary?

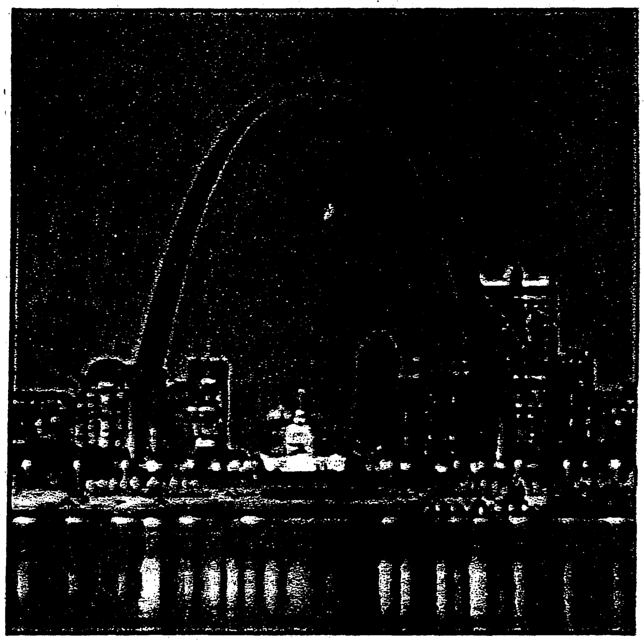

Figure 5: The Gateway Arch.

Conclusions

We can do our science and engineering colleagues (and incidentally ourselves) a favor by convincing students that simple standard models in math classes are standard because of striking **quantitative** agreement with physical reality. The simplest and most visually compelling way to do this is by extracting data from a broad range of visual images.