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The Dynamical Systems and Technology Project

1. Introduction. One of the things that has bothered me most during my career as a mathematician is the fact that most people have no clue as to what is happening in contemporary mathematics or why this field is important. Most high school students I meet think mathematics ended in the days of Pythagoras and Euclid; my calculus students think that Newton and Leibniz put the finishing touches on mathematics; many high school teachers that I work with have no contact whatsoever with anything that is new and exciting in mathematics; and many mathematics majors in college graduate without ever having seen any twentieth century mathematics, not to mention twenty-first century mathematics. I compare this ruefully with other fields of science, where every student seems to be aware of the great progress in the human genome project, the remarkable advances in computer technology, the spectacular discoveries in astronomy, and on and on in every other field of science. In contrast, we in mathematics have designed our curriculum linearly, so that few students experience contemporary ideas in this field. This, in my opinion, is most unfortunate for the field of mathematics, and for the country as a whole, as fewer and fewer American students pay any attention at all to careers in the mathematical sciences.

The main goal of the Dynamical Systems and Technology Project (DS&TP) is to help change this culture. The specific aim of the project is to provide a variety of opportunities for students in introductory mathematics and science classes to see that mathematics is an alive and vibrant field in which there are many open questions that even they, at their tender age, can begin to contemplate and investigate.

My field of research, discrete dynamical systems, offers an ideal opportunity to involve students in modern mathematics. This field has the advantage of being quite accessible to lower level students (much of my research involves iterating simple quadratic or exponential expressions — topics I can easily explain to students). As such, it relates perfectly to many facets of the lower level mathematics curriculum, especially algebra, geometry, calculus, and differential equations. Moreover, this field is replete with stunningly beautiful images such as the Mandelbrot and Julia sets, fractals, and strange attractors. Students who see these images cannot help but marvel at their complexity. My experience has been that, when they realize that these images relate directly to what they are currently studying in their mathematics classes, they become not only intrigued, but also genuinely excited about mathematics. This, to my mind, is just what the discipline needs.

In the early 1990s, with support from the National Science Foundation, I organized after school "chaos clubs" in the Boston Public Schools. This was an attempt to show inner city teachers that the inclusion of modern mathematics in standard mathematics courses would serve to excite and motivate students. The events we organized drew many students and teachers and were quite successful. This led me to organize Mathematics Field Days at Boston University. These events now draw over 1,000

New England students and teachers to campus each year for a daylong introduction to contemporary mathematical ideas.

2. The DS&T Project. The success of these outreach activities led me to organize the DS&TP in order to bring these activities to a much wider audience. With NSF support, this project brings a number of the chaos club activites online. Among the activites at the DS&TP website (`http://math.bu.edu/DYSYS`) are:

- A series of 14 java applets for investigating topics related to chaos and fractals. These applets include several challenging games and fractal movie-making utilties;
- An online series of 9 explorations that introduces the mathematics and graphics behind the Mandelbrot and Julia sets;
- An interactive resource that describes the mathematical topics included in Tom Stoppard's recent play *Arcadia* for use in mathematics classes for liberal arts students;
- A variety of fractal movies that show many aspects of how the fractal chaotic regimes of dynamical systems explode as parameters change.

More applets and explorations are in devlopment. While I do not have the space to describe all of the activities available on the DS&TP website, here is a brief description of four of these activties.

3. Fractalina. Fractalina is a java applet designed to introduce the concept of an "iterated function system." Here is the simplest example of such a system, sometimes called the "chaos game." Put three points at the vertices of a triangle in the plane. Color one vertex red, one green, and one blue. Take a die and similarly color two sides red, two sides green, and the remaining two sides blue. Now choose any point whatsoever in the triangle as the starting point and roll the die. Depending upon which color comes up, place a new point half the distance to that vertex. Then repeat this process over and over, placing a new point after each roll halfway between the previous point and the appropriate vertex. When the die is rolled thousands of times the resulting image is a surprise: It is not a random mess as most first-time players expect. Rather, the resulting image is one of the most famous of all fractals, the Sierpinski triangle (Figure 1).

Note that the resulting image basically has three self-similar pieces, each of which is half the size of the original. These are the same numbers that we used to generate the image, indicating that we can "go backwards." That is, we can read off the rules to produce this image from the geometry of the resulting fractal image. This is where the mathematics and applications of this subject lie.

Fractalina allows the user to experiment with this and many other chaos games. As another example, starting with six vertices at the corners of a regular hexagon

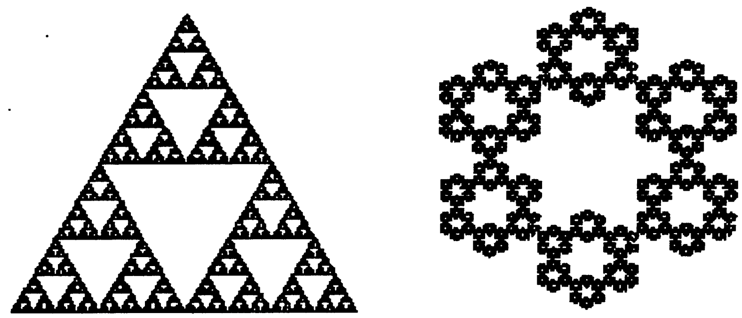

Figure 1: The Sierpinski triangle and hexagon.

(so no colors on the die), and compressing distances by a factor of 1/3 (rather than 1/2), yields the Sierpinski hexagon, a figure with six self-similar pieces each of which is 1/3 the size of the original. There are a number of variations of the rules that can be included. For example, we can combine compressions and rotations about the vertex. The famous Koch snowflake curve is constructed using four such rules. And the variations are endless, yet in each case the students can go backwards: by examining the geometry of the resulting fractal, the students can reconstruct the rules that generated it. In order to figure out these rules, students must become familiar with the geometry of affine transformations, a subject that is not included very often in the secondary or college mathematics curriculum. I have used these ideas everywhere from middle school lectures to sophomore linear algebra courses in college with great success, and I am sure that the interactive on-line version works very well in all of these settings.

4. Fractanimate. We have also developed an applet that allows users to string together a collection of fractals derived from different chaos games in a movie. In the chaos club, I challenge students to make a movie that is both "beautiful" and that I cannot figure out how they made it. Of course, beautiful here means with a lot of underlying symmetry, so there really is a lot of geometry in this project. Students often spend hours making these animations (several are posted on the DS&TP website).

5. The Chaos Game Challenge. In order to help students and teachers understand how iterated function systems work, we have developed a real "chaos game." The aim of this game is to teach students the "addressing" ideas that produce these fractal images. We give the students the outline of the Sierpinski triangle down to some level, say the third level, so that there are 9 triangles visible, or the fourth, where there are 27 triangles. One little triangle is randomly shaded: this is the target. We also give the students a starting point, usually the lower right-hand vertex. Then the idea is to move the starting point into the interior of the target via the moves as in Fractalina. It turns out that there is a minimum number of moves (at a particular level) in which this can be accomplished independent of where the target is situated. The challenge is to figure out this algorithm. This is by no means easy. Students can often figure out how to hit a given target, but they have a tremendous amount of difficulty seeing the big picture and determining an algorithm that works for any target at any level. We have seen students in the chaos club work for weeks to figure out this challenge. I have used this activity with students ranging from sixth grade to senior year in college (and even in my workshops for college faculty). Believe me, figuring out the algorithm is not easy!

6. The Mandelbrot Set Explorer. The field of complex dynamics has received much attention in the past twenty years. Unfortunately, much of this has been confined to the beautiful computer graphics images of the Mandelbrot and Julia sets, without much attention paid to the equally beautiful mathematics that lies behind these images. In this series of nine interactive explorations, students are introduced to such concepts as the relationship between the Mandelbrot and Julia sets, rotation numbers of the primary bulbs, and bifurcations. Since the only mathematics required to understand these ideas is the geometry of complex numbers, these exporations are accessible to students at the advanced high school level and above. Each exploration features a gentle introduction to the mathematical idea, contains numerous graphics images and animations illustrating the concept, and allows the user to experiment on his or her own with various applets.

7. Role of Technology. One of the main reasons for the explosion of interest in dynamical systems in the 1990s is the fact that you simply cannot investigate many dynamical systems without the use of the computer. Producing the Sierpinski triangle and hexagon above requires hundreds of thousands of iterations of the rules. Generating images such as the Mandelbrot and Julia sets requires millions of iterations. This is the reason why the Mandelbrot set was not seen until 1980, despite the fact that this image is the parameter space for the simplest possible nonlinear iteration, namely the family $z^2 + c$.

Thus, to expose students to these contemporary ideas in mathematics, you need to provide them with readily available, easy-to-use tools with which to investigate and experiment with dynamical phenomena. This is the guiding philosophy behind the DS&TP website: each tool serves a single purpose and allows the user to delve into a particular mathematical topic of interest. Unlike many of the research tools available, these applets permit the student and teacher to become involved in the mathematics at the outset, with very little preparation time necessary to use the software. Better yet, the software is free, runs on all platforms, and comes with detailed background information.

8. Participants. Numerous students and colleagues have contributed both ideas and software to this project. The list includes undergraduates Noah Goodmann, Rodin Enchev, Lauren Giordano, Chris Mayberry, Yakov Shapiro, Lisa Smith, and Johanna Voolich; graduate students Clara Bodelon, Monica Moreno Rocha, and Adrian Vajiac;

high school teachers Jon Choate and Alice Foster; and college faculty James Denvir (Marshall) and Kevin Lee (St. Catherine's).

9. Field Testing and Impact. As required by the NSF grant funding the DS&TP, over 75 college and high school teachers have served as field-testers for various portions of the site. All received training at preliminary workshops, and all submitted lengthy reports detailing their students' experiences using the website.

As of September, 2002, the website has received well over 395,000 individual hits. In addition, I have delivered over 500 lectures on these and related materials since 1993. While some of these were aimed at research mathematicians, many others involved high school or college students and faculty. In all of these talks I routinely use the tools available at the DS&TP website to introduce and explain modern topics in dynamics.

10. Publications Related to DS&TP:

1. Explorations in the Chaos Club. Focus. **15** (1995), 8-9.
2. Putting Chaos into Calculus Courses. In Discrete Mathematics in the Schools. DIMACS Series in Discrete Mathematics. Amer. Math. Soc. (1997), 239-254.
3. Chaos in the Classroom. In Designing Learning Environments for Developing Understanding of Learning Geometry and Space, Erlbaum Associates (1998), 91-104. `http://math.bu.edu/DYSYS/chaos-game/chaos-game.html`
4. The Fractal Geometry of the Mandelbrot Set: I. Periods of the Bulbs. In Fractals, Graphics, and Mathematics Education. MAA Notes **58** (2002), 61-68. `http://math.bu.edu/DYSYS/FRACGEOM/FRACGEOM.html`
5. The Fractal Geometry of the Mandelbrot Set: II. How to Add and How to Count. Fractals **3** (1995), 629-640. `http://math.bu.edu/DYSYS/FRACGEOM2/FRACGEOM2.html`

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