Discovering Geometry
Using *Geometer's Sketchpad*

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ABSTRACT

Geometer's Sketchpad is an easy-to-use dynamic construction and exploration tool for use in a variety of classes. Using *Sketchpad*, students discover patterns, formulate conjectures, and verify their conjectures by creating small examples. Students then generalize their justifications to prove a geometric relationship. This paper presents two examples of geometric explorations using *Sketchpad*.

INTRODUCTION

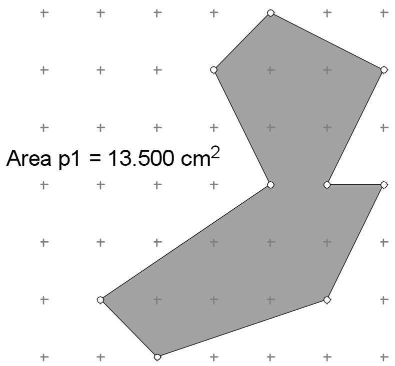
Geometer's Sketchpad is a dynamic construction and exploration tool that enables students to explore and understand mathematics in ways that are not possible with traditional tools. *Sketchpad* encourages a process of discovery in which students first visualize and analyze a problem, then make conjectures before attempting a proof. Since *Sketchpad* is easy-to-use, students are focused on the exploratory activity.

Geometry courses for students intending be teachers at any level (elementary through high school) are ideal courses in which to integrate *Geometer's Sketchpad*. The software is easy to use, appropriate for a discovery approach to understanding geometric ideas and concepts, and encourages creativity, deeper insight, and cooperative work in problem solving. This software makes it possible to display and perform calculations on geometric objects in a manner that would be time consuming with pencil and paper. Using this program to explore geometric relationships and formulate conjectures is in line with the National Council of Teachers of Mathematics Standards.

The following *Geometer's Sketchpad* investigations that I have adapted and developed explore Pick's Theorem and Napoleon triangles. In each activity, students are not given the theorem statements or properties, but rather are asked to discover relationships. Once students determine the geometric relationships, they are asked to write a theorem statement and eventually provide a mathematical proof.

INVESTIGATION OF PICK'S THEOREM

In discovering Pick's theorem, students use *Geometer's Sketchpad* to construct a polygon (they first select the Show Grid option and Hide Axes option from the Graph Menu). Secondly, they construct the polygon interior and measure the area of the polygon. Next, students count the number of points of the polygon that lie on the border, *b*, and the number of points on the interior of the polygon, *i*. Then students record the values for *b, i,* and *Area* for the polygon constructed. An example is given below.



	b = number of boundary points	i = number of interior points	Polygon Area
Polygon 1	9	10	13.5
Polygon 2			
Polygon 3			

Next they change the shape of the polygon and record the new values corresponding to this polygon. Students continually change the shape of their polygon and/or change the number of vertices of their polygon and record the new entries in the table until they see a pattern. Once a pattern is observed, students are asked to write a well-structured sentence of their discovery.

The more challenging part of this investigation is for students to provide mathematical reasons, which support Pick's Theorem. Using *Geometer's Sketchpad* to guide their thinking, students investigate why their theorem statement is valid for a triangle with unit base and unit height. Next they try to understand the theorem for different sized triangles. After understanding the theorem for triangles, students investigate Pick's Theorem using rectangles. Finally, they generalize their mathematical reasoning for any polygon.

DISCOVERING NAPOLEANIC TRIANGLES

This activity is attributed to Napoleon Bonaparte and adapted from a Key Curriculum Press Activity. Napoleon Bonaparte was a dictator and emperor of France and was also interested in mathematics. His discovery was: If you take any triangle and draw equilateral triangles on the sides, then connecting the incenters of the three equilateral triangles forms an equilateral triangle. Other interesting relationships from this construction can also be discovered. Students are given the following construction steps and questions to investigate.

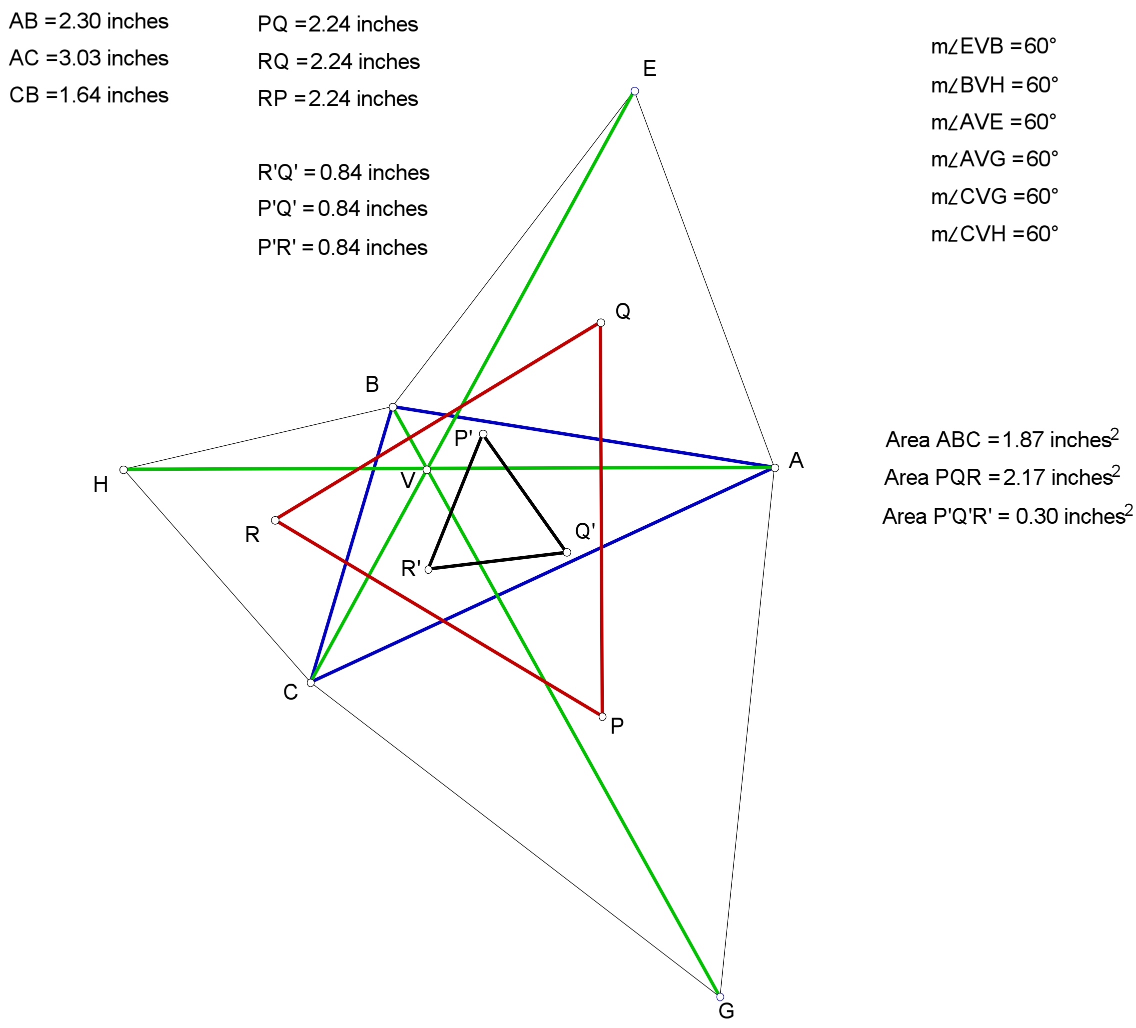
Construction steps:

1. Construct a triangle ABC. Color the side lengths blue.
2. Construct equilateral triangles on the sides of the original blue triangle.
3. Connect the incenters of these triangles to form triangle PQR. Color these three segments red. This triangle is referred to as a Napoleonic triangle.
4. Connect the outside vertices of the equilateral triangles with the opposite vertex of the original blue triangle. Color these line segments green.
5. Reflect each vertex of the red Napoleonic triangle over the closest edge of the original blue triangle. Connect these three points to form triangle $P'Q'R'$.

Questions to investigate:

1. What is true about the red Napoleonic triangle, PQR ?
2. What is true about the green line segments?
3. The three green segments intersect at a point. How is this point related to the original triangle?
4. What is true about the triangle $P'Q'R'$?
6. How does the area of the triangle $P'Q'R'$ compare with the area of the red Napoleonic triangle PQR ?

The construction, together with some calculations, is shown below. After students have followed the constructions they move the vertices of the original red triangle PQR to investigate relationships and to form conjectures. This activity forces students to understand the construction techniques for creating an equilateral triangle and finding an incenter. Additionally, students must apply several geometric properties and theorems when they verify their discoveries.



CONCLUSION

Since students have not seen either of these theorems in a previous course, I have found great success in using these particular activities. Students are more willing to explore and conjecture when it is a "new" theorem. Students find that the geometry course requires critical thinking and allows for creativity. They also discover that they need to bring an open-minded, energetic and positive-thinking attitude to class when they discover geometric relationships on their own using *Geometer's Sketchpad* as a tool. I believe that the use of *Geometer's Sketchpad* helps students challenge their existing notion of what mathematics is about.

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