"ENLIGHTENING THE BLACK BOX: TECHNOLOGY MOTIVATING MATHEMATICS"

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Introduction Possibly referring to calculators such as the TI-92 or Casio CFX-9970G, Usiskin (1999, p. 1) relates: "[U]ser-friendly calculators have appeared that can solve literal algebraic equations, manipulate algebraic expressions, differentiate and integrate and solve systems of equations....[and which] force an examination of the amount of paper-and-pencil mathematics a person needs in algebra through calculus and beyond." Usiskin raises a number of questions, including "What new understandings, if any, can arise from calculator use, and what understandings, if any, may be lost?" The focus of this paper is on the understandings that arise that often relate back to some of the very understandings many consider in danger of being lost!

An example commonly cited is that a graphing calculator forces students to have greater than usual awareness of domain and range by having to explicitly enter window viewing settings. NCTM (2000, p.299) claims that, with technology, "students can easily explore the effects of changes in parameter as means of better understanding classes of functions [e.g., ax^2 + bx + c]." There are even more direct examples of how mathematics is specifically motivated by the use of technology.

Line of best fit, interpolating polynomials, and complete graphs provide three relevant opportunities for viewing technology and mathematical theory as partners rather than as competitors, a view with great implications for the current dialogue on mathematics education reform. The goal of using technology with understanding often forces us to reflect upon the underlying mathematics and often leads to lesser known ways of attaining this goal without needing as highly-powered mathematics as is often assumed. Let us examine three topics which are also discussed by Lesser (1999b) and are utilized to some degree in certain progressive college algebra curriculum materials (e.g., Mayes and Lesser 1998).

Example One: Polynomial Graph and Roots. A student can more effectively utilize a graphing calculator to graph functions by applying a theoretical result (which is easy to use and whose proof requires only the Factor Theorem and the triangle inequality) to ensure all interesting behavior of a function (e.g., roots, turns, inflection points) is within the rectangular viewing area. The result states that the x-coordinates of all interesting behhavior will be in view if the window is of the form ±M, where M is the larger of 1 and P/Q, where P = the sum of the absolute values of all coefficients except the lead coefficient and Q = the absolute value of the lead coefficient. For example, applying the result to the polynomial y = 6x^3 - 2x^2 + x - 15 tells us that the x-coordinates of all roots and turns must be between ± max(1, 18/6) = ±3. Even teachers who teach without technology could find this result useful. Such a traditional teacher would likely cover the Rational Roots Theorem and his students would surely appreciate that the above interval in this case would eliminate 8 of the 24 candidates for rational roots identified by the rational root theorem!

Example Two: Interpolating Polynomial. When a CAS (Computer Algebra System) or graphing calculator crunches an interpolating polynomial (a polynomial that passes through all n ordered pairs in a data set), a student may do the same by hand using the intuitive, easily generated Lagrange "factored" form, which generates a polynomial of degree equal to (or less than) n-1. Their equivalence could be verified by hand or by using the "Simplify" command of a CAS such as Derive. So rather than simply accepting the technology-provided interpolating polynomial at face value, students can follow the recommendation of NCTM (2000, p.297) that they "become fluent in performing manipulations by appropriate means -- mentally, by hand or by machine....to generate equivalent forms of expressions or functions, or to prove general results."

Example Three: Line of Best Fit. When the computer outputs a line of best fit, a student may derive the formulas involved using algebraic technique of completing the square (the same technique used in deriving the quadratic formula and in deriving the formula for the vertex of a parabola) instead of using calculus tools such as derivatives. As Lesser (1999b, p. 783) states, "Ironically, the proof technique of completing the square has been a standard topic that many teachers fear will be displaced by technology. We have perhaps offered a justification for keeping this topic in the curriculum in a way that supports, rather than competes with technology." Lesser (1996, 1999b) reminds us that it is pedagogically wiser and mathematically simpler to start with a "one-parameter" linear model that is forced to go through the origin.

We not only need to reflect on the mathematics to better understand the technology, but can also use the technology to reflect upon the mathematics. Lesser (1999a) presents a sequence of explorations and responses to student questions about the criterion “minimize the sum of the squares of the vertical deviations between the fitted line and the observed data points” and concludes that a noncalculus-based motivation is more feasible than is often assumed for each aspect of this. This is supported, for example, by the use of dynamic-geometry software Cabri by Ehnert (1999) to show that the sum of the absolute values of the errors may remain unchanged when the proposed line of fit is moved within a certain range. This demonstrates the undesirable nonuniqueness pitfall that often happens when using the criterion of minimizing the absolute errors.

Conclusion In our primary examples we have seen a variety of “traditional” mathematics content and tools (e.g., Factor Theorem, triangle inequality, factoring, and completing the square) that are not replaced by but can actually be (re)motivated by the introduction of technology into the classroom. As Lesser (1999b, p.783) states, “Let us seek opportunities to connect tradition and reform, calculation and concept, and the many representations that build a rich mathematical experience.” Hopefully, we have pointed out some of the ways in which technology itself motivates some of these opportunities and we encourage others to identify additional examples.

References

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