ASSESSMENT: DOES THE PUNISHMENT FIT THE CRIME?

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"My object all sublime
I shall achieve in time--
To let the punishment fit the crime--
The punishment fit the crime;
Gilbert and Sullivan - The Mikado

Abstract

Assessment has various purposes. Is it for grading and sorting students? Is it for encouraging learning? The answer is yes to both, but when both technology and students' skills are evolving so rapidly, then assessment style must also evolve to ensure it continues to fulfill these objectives. In this paper we describe some of our experiences with changing assessment styles during the ongoing T-TIME project (http://www.shu.ac.uk/maths/).

Introduction

Assessment (a *punishment* for both the student and the lecturer!) manifests itself in many ways according to the nature of the stated learning outcomes, and what it is that the students actually *do* as a result of what we put in front of them (what we might call their *crimes*). It is critical to make sure that the assessment activities we devise actually match what we say we want the students to have learned. It is particularly necessary to keep this practice under review at a time when rapidly evolving technology is enforcing change in the way that mathematics is taught and learnt.

We have written previously [e.g. 1, 2, 3] about our experiences with forms of assessment other than the traditional examination, including investigations, case studies and learning diaries, with a view to exploiting fully the possibilities in mathematics of assessing the full range of student activity, including key transferable skills. It still does seem however that within the mathematics community, the traditional examination is dominant.

In this paper we explore these aspects further. We are still asking what "product" the students should be offering us to demonstrate their mathematical competences. What assessment criteria address the various processes of “doing mathematics” now? We argue that one part of mathematical practice that is essential to our students, includes being aware of the “audience” to which you are demonstrating your skill.

How could we assess?

At its root, the assessment system is based on the generation of evidence by students which shows that they can achieve some preset, often national standards. We as examiners may make judgements about a student's competence from a variety of sources. Some examples were mentioned earlier, and one possible broad categorization is as follows:

- Performance evidence:

  This could include direct observation, samples of work, a project, a simulation of an activity, or indeed an examination!
- Indirect and supplementary evidence:

  This could include oral questions and answers, written tests of knowledge of process, reports or testimonials from a teacher or mentor, or indeed the student, evidence from previous or outside achievements and performance.

How do we as a mathematical community exploit this range of possibilities? In the main we continue with examinations, but this raises fundamental questions concerning the nature and purpose of higher education. If we worked in an elite system in which we were simply required to propagate ourselves, then assessing only the narrow academic knowledge and skills would perhaps be justifiable, although even then it is doubtful what value there is in producing mathematicians with lamentable communication skills [4]. However, we work in a system of mass participation, in which most students of mathematics can benefit from the disciplined thinking skills of mathematics, but do not call themselves a mathematician. Thus we should acknowledge that we need to develop their skills so that they can address a wider audience.

What is the audience that the student is addressing?

We are suggesting that the way mathematics is done, as a human activity, depends on the audience to which it is to be useful – this assumes that "doing mathematics" in this context includes communicating the results suitably. For example, the people to whom this paper is addressed will be mainly teachers and lecturers of mathematics. Our students are possibly some of the best students of mathematics in the world, although from the rather superior and punitive tone adopted in certain email "conversations" it is apparent that some may have a problem accepting this!

The reader of this paper must surely concede that most people in the "real" world in which we work have far less mathematical knowledge and expertise than either ourselves, or indeed our students, and that most of our students will go on to work in that relatively unmathematical world. Therefore a critical skill in any student's mathematical education is that having understood and solved problems, understood the solutions and convinced themselves that they are correct, they have then the onerous task of choosing the vehicle suitable for explaining their correct solutions to their particular audiences.

It is thus necessary to look at a solution in conjunction with the range of the audience. A solution to any given mathematical problem can be straightforward, complex, visual, numerical, a persuasive argument, a proof, an algebraic manipulation, and so on, with the range of possibilities here considerably enhanced by technology. The mode of solution needs to be chosen to enable the audience to understand it, to obtain information and conviction about it, and to have the possibility of sharing ideas.

The "mathematician" must be aware that the audience could include people familiar with the subject who know them, people familiar with the subject who do not know them; people not familiar with the subject who know them, and people not familiar with the subject who do not know them. Unfortunately the last category tends to be very common, and the one which is least addressed in academic courses.

Pause for thought - an example

Here we pause for a while to look at a particular problem that we will use to illustrate some of our practice in respect of the ideas above. First here is the specification for this example, as given to the students.

Selling Burgers

Suppose that you have some friends who run a local fast food outlet and they have just completed a trial for a new product, the `Chilliburger'. They ask you to help them to make some decisions based upon their trial.

The market research trial suggests the following results.

Shop's selling price of a `Chilliburger' ($)	*Quantity sold per month*
0.50	*750*
1.0	*560*
1.50	*470*
2.00	*320*

Input the data from the table and perform a linear regression to give the quantity sold as a function of the selling price. Write down this linear regression function.

Plot the data points and the regression function on the same graph, choosing a graph that shows all the points and the y-intercept. Sketch what appears on the screen, and label the axes of' your sketch appropriately. Comment on how well the model fits the data

Interpret the meanings of x- and y-intercepts and the gradient of the regression line for your friends, in terms of prices and customers.

Use your model to predict the demand for `Chilliburgers' at prices of $0.80 and $1.25.

Your friends have been given information from the suppliers of 'Chilliburgers'. The shelf-life of the product is such that your friends have to buy monthly. You and your friends add on mark-up costs to give a realistic selling price for different quantities bought from the supplier each month.

Quantities Supplied, per month	*Selling price in $*
100	*2.10*
200	*1.50*
500	*1.00*
800	*0.90*
1000	*0.80*

Write down this linear regression function. For what range would you consider the model to be unreliable?

Your friends want to know when the quantity demanded at a given price would equal the quantity they could supply at that price.

Explain and show how you would find this algebraically, numerically, graphically and with a suitable persuasive argument.

Write a report commenting on the advantages and disadvantages of all of these methods in terms of accuracy, speed, skills needed by yourself and your friends to understand each of the solution methods and suitability for various audiences. You might like to include a tabular summary of your conclusions somewhere in your solution.

Sketch the two linear models on the same graph and indicate ranges for which

demand > , = ,< supply.

Your friends are rather careful and do not like waste and will not be happy if they do not sell all their Chilliburgers each month. Based on your model what would you recommend?

Write in the form of a letter any further advice for your friends on their pricing and buying before they start out on their new venture. Show carefully in this letter how you would communicate all your ideas to them.

Comments

What is traditionally regarded as “the mathematics” is straightforward (for us!) including only some least squares fitting and solution of simultaneous equations. However this is not the only thing the question is demanding. An essential ingredient is the communication of the solution in the appropriate form for the considered audience. Some help is given in the question to achieve this.

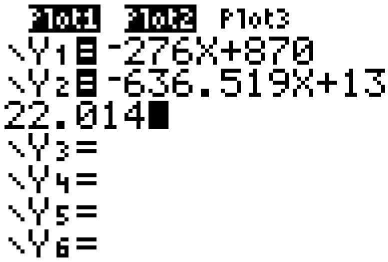
Note the problem contains many words and very few symbols, which raises the question of how much the solution should reflect the question style. The student when answering questions must consider what evidence they are going to provide in order to fulfill what the question is specifying. In making judgements about what evidence to provide for the solution, from whatever viewpoint, the assessor of the work must be convinced that it satisfies a number of key criteria:

- the evidence must be *valid* - it must be correct!
- the evidence must be *authentic* - it must be clear that it is each student's own work (this is always a difficult thing to judge - in an ideal and unconstrained world, one would observe students working)
- the evidence must demonstrate that performance is *consistent* and that skill competence is still *current* (not just a lucky solution and not relying on archaic skills)
- the overall evidence must be *sufficient* to demonstrate competence across the full range required.

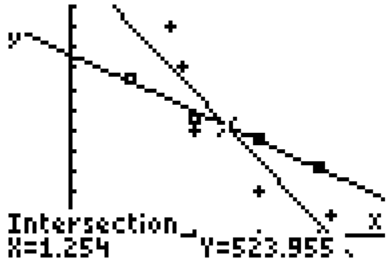
The students must consider all of these checks before presenting their solutions as well as addressing the range of their audience.

Here are some typical solution screens for the "mathematics".

Least squares fit on a TI-83 gives



The graphs are

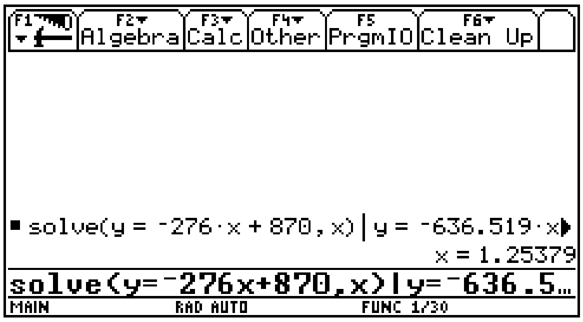


Tables - (numbers)

X	Y1	Y2
0.000	870.00	1322.0
.200	814.80	1194.7
.400	759.60	1067.4
.600	704.40	940.10
.800	649.20	812.80
1.000	594.00	685.50
1.200	538.80	558.19

X=0

Symbolically (cheating!)



This really emphasises how much power flows from the technology in the purely technical side of getting and displaying the “answers”. However getting an answer is only the start of convincing both ourselves and the various others who may be interested.

The question above shows how we are trying to use the students’ communication skills to change their attitudes when solving problems. We are trying to instil the idea that they have to communicate over some range of audience. Most people will not have the student's own mathematical ability. Therefore it is imperative that they examine all possible ways of communicating their own valid conclusions in an effective way. A single solution technique is not a complete answer when students must also consider how others will relate to and understand the mathematical problem. The reader might notice that we use a development of the SONG approach [3], blending symbolic, oral, numerical and graphic approaches. This incidentally also provides checks on the answers!

Conclusion -how do we assess the *whole* process of "doing" mathematics?

Briefly, we have changed some aspects of our assessment practice to adjust to the modern day needs and skills of students. A different approach is necessary to assess these different skills, which are often neglected in many mathematical courses. A typical comment is "I came here to do mathematics, not to write letters and essays". We have reported on this elsewhere [e.g. 1].

We did start with a quote from Gilbert and Sullivan:

"My object all sublime, I shall achieve in time,
To let the punishment fit the crime, the punishment fit the crime"

It seems the crime - that is, what the student is doing - must be punished, and assessment really can feel like a punishment, for setter, marker - and student! The added factor here is that the range of both the crime and those affected by it must be taken into account properly when devising the punishment.

Appropriate assessment can help us to broaden a student's view and to overcome the preconceptions which often include a narrow view of mathematics, that they may have gained both from previous education and from society at large. We act as role models, and it is up to us to show them good habits in approach to problem solving, and tool selection, or else they will try to cut their potted plant with a combine harvester.

We have given one example of how an emphasis on communicating a solution can help to develop robust practice using various methods for the solution of problems. We are pleased to share our thoughts with our colleagues. Amongst other aims, we are working towards changing public perception of mathematics, and mathematicians, away from the nerd image, to that of a full and participating member of the human race [4 again]!

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