ISSUES ON PROBLEM SOLVING: DRAWING IMPLICATIONS FOR A TECHNO-MATHEMATICS CURRICULUM AT THE COLLEGIATE LEVEL

By Auxencia A. Limjap, Ph.D.[1]
Mathematics Department
De La Salle University
2401 Taft Ave., 1004 Malate
Metro Manila, Philippines

ABSTRACT

This paper presented various issues about pedagogical and cognitive aspects of problem solving and explored ways to lessen the heavy cognitive load of a problem solving task. It established a problem type schema for students at different levels. It recognized the role of modern technology as a cognitive tool that promotes learning mathematics with understanding. It designed the framework of a techno-mathematics curriculum for algebra at the collegiate level.

INTRODUCTION

As modern civilization requires relentless quantification and critical evaluation of information in daily transactions, it becomes necessary to develop newer ways of thinking and reasoning that can be used to learn and do mathematical activities. Through problem solving for instance, we acquire a functional understanding of mathematics needed to cope with the demands of society.

School mathematics of the twenty first century is viewed by educators to be that which should engage a learner in problem solving and reasoning. It should also foster deep understanding and develop the learner's critical and analytical thinking. Instruction should not be limited to plain mastery of algorithms or the development of certain mathematical skills. It should involve learners in investigation through "exploring, conjecturing, examining and testing" (NCTM, 1990, p.95). It should be tailored to promote reflective thinking among students.

A wealth of research on mathematics education and cognitive science in the last decade has dealt with the pedagogical and cognitive aspects of problem solving. Rivera and Nebres (1998) note specifically "the numerous published research studies of Fennema and Carpenter on Cognitively Guided

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[1] During the presentation of this paper, the author is doing research at the Ohio State University as a Fulbright Visiting Scholar. She is with the Mathematics, Science and Technology Education, College of Education, School of Teaching and Learning, 333 Arps Hall, 1945 North High St., Columbus, OH 43210-1172.

Instruction (CGI) in the last quarter of this century [which] point to the pernicious effects of status quo ways of thinking about mathematics and problem solving (i.e. existing mathematics culture)"(p.11). CGI recognizes the "acculturation of school children to an algorithmic approach to learning basic arithmetical facts" which pervade the current school mathematics culture and which have been proven to be "detrimental to children's own ways of thinking about problem solving and computations" (p.12).

Bishop (1999) adds that "research has shown the importance of the idea of situated cognition which describes the fact that when you learn anything you learn it in a certain situation" (p.41). Thus for learning to become meaningful, the learner has to actively participate in the formation of mathematical concepts. She should not passively receive knowledge from an authority but should be involved in the construction of knowledge.

The theory of active construction of knowledge influenced many learning theories formulated by staunch contemporary mathematics educators like Von Glasersfeld, Cobb, Bauersfeld, Vygotsky and numerous others (Rivera, 1999). In fact, "problem solving and mathematical investigations based on a constructivist theory of learning, have been the main innovations or revivals for the last decade" according to Southwell (1999, p.331).

Willoughby (1990) believes that the abundant books, pamphlets and courses on critical thinking and problem solving that have been propagated in the 1980s cannot be of help unless certain pedagogical misconceptions are clarified. This includes prescribed rules such as finding key words in a problem to decide the appropriate operations on the values given in the problem, or applying arithmetic algorithm to any word problem. Developing critical and analytical thinking through problem solving takes time and a lot of teacher's commitment and dedication. (Willoughby, 1990; Barb and Quinn, 1997).

Developing critical and analytical thinking involves pedagogical conceptions with a philosophical basis. This paper adheres to the constructivist theory of learning and promotes the belief that problem solving processes rest on basic thinking skills which are best developed within a constructivist framework.

Another challenge of the new millenium is the proper use of the ever advancing technology in education. Researchers have to look into the quality of instruction and curriculum which utilize technology. Educational technology should be guided by pedagogical principles that guarantee effective learning, and not subordinated to technological ends. Thus, "technology should be used to advance educational programs, [and] should not determine programs" (Witt, 1968, p. 145). How to empower students further in learning with the use of technology should be the concern of curriculum designers.

In the light of existing literature base on mathematics instruction and flourishing research studies on mathematics teaching and learning, this paper explores issues and finds ways of fostering critical and analytical thinking through problem solving. Then it draws implications regarding the design of a techno-mathematics curriculum for algebra at the collegiate level that establishes problem type schema. This design is supported by a philosophical basis of the role of technology in the acquisition of mathematical knowledge. The design is not instrument specific, since it is intended to be adaptable to whatever technology is available to both teachers and students be it in progressive countries or in the third world countries.

THE LEARNER AND COGNITIVE PROCESSES

Recent research studies on mathematics education have placed its focus on the learners and their processes of learning. They have posited theories on how learners build tools that enable them to deal with problem situations in mathematics. Blais reveals that

> the philosophical and theoretical view of knowledge and learning embodied in constructivism offers hope that educational processes will be discovered that enable students to acquire deep understanding rather than superficial skills. (Blais, 1988, p.631)

As learners experience their power to construct their own knowledge, they achieve the satisfaction that mathematical expertise brings. They acquire the ability to engage in critical and analytical context of reflective thinking. They develop systematic and accurate thought in any mathematical process.

O'Daffer and Thorquist (1993) define critical thinking as "a process of effectively using skills to help one make, evaluate and apply decisions about what to believe or do"(p.40). They cited the observations of Facett(1938) on a student using critical thinking as one who

1. Selects the significant words and phrases in any statement that is important and asks that they be carefully defined;
2. Requires evidence supporting conclusions she is pressed to accept;
3. Analyzes that evidence and distinguishes fact from assumption;

4. Recognizes stated and unstated assumptions essential to the conclusion;
5. Evaluates these assumptions, accepting some and rejecting others;
6. Evaluates the argument, accepting or rejecting the conclusion;
7. Constantly reexamines the assumptions which are behind her beliefs and actions.

Critical thinking abilities can only be developed in a setting which the learner has ample knowledge and experience. Thus, fostering critical thinking in a certain domain entails developing deep and meaningful learning within the domain.

Learners can acquire critical thinking strategies by using what cognitive and developmental psychologists call a cognitive schema. Smith, Knudsvig and Walter (1998, p.50) describe a cognitive schema to be “a scheme, method, process by which (one) can see, organize and structure information” for better comprehension and recall. Through the schema learners interpret, analyze, organize and make sense of every information given in a problem situation through a constructive process called reflective abstraction.

Through reflective abstraction, critical thinkers are able to assimilate information into their mathematical network and build from their prior knowledge. They can accommodate new ideas including those that conflict with what they know or believe and negotiate these ideas. They are willing to adjust their belief systems after reexamining information. They are also able to generate new ideas based on novel ideas that are available to them. They are expert problem solvers who can handle abstract problem information and make sense of different problem situations.

On the other hand, novice problem solvers are not able to handle abstract mathematical concepts. They have difficulty recognizing underlying abstract structures and often need to make detailed comparisons between current and earlier problems before they can recognize the abstract information in the solution of the current problem (Reed ,1987; Reed, Dempster, Ettinger, 1985; Anderson, 1984; Ross, 1987, as cited by Bernardo, 1994). They usually resort to algorithmic activity and not to the perception of essence. Blais (1988) observed that “they resist learning anything that is not part of the algorithms they depend on for success”(p.627). They tend to be very shallow in dealing with problem situations because of the lack of depth in their experiences while engaging in mathematical activities.

All problem solvers, whether experts or novices, develop a cognitive schema which cognitive scientists call problem-type schemata when confronted with a mathematical problem. According to Bernardo (1994), "[k]nowledge about the problem categories include information about the relevant underlying principles, concepts, relations, procedures, rules, operations and so on"(p.379). Further, he adds, "problem-type schemata are acquired through some inductive or generalization process involving comparisons among similar or analogous problems of one type"(p.379). Learners represent, categorize and associate problems to be able to determine the appropriate solution. The expert's schematic processing leads to an accurate analysis of the problem which the novice hardly achieves.

Bernardo (1994) claims that "the novices' schemata (expectedly) include[s] mainly typical surface-level information associated with a problem type, whereas experts' schemata include[s] mainly statements of abstract principles that [are] relevant to the problem type"(p.380). One example of the difference in the processing of experts and novices given by Blais (1988) is on their reading process of a mathematical material. Blais (1988) observes that,

> [w]hen novices read, the process almost always appears to be directed toward the acquisition of specific information that will be needed for algorithmic activity, (whereas) the reading process used by experts is directed toward the perception of essence. (p.624)

Experts seem to readily categorize the mathematical information in the material being read, thus facilitating the processing of information that lead to the correct solution. They are able to attain some sort of a visual form of say an algebraic expression and are able to communicate this before they perform the algorithmic activity. Besides, they can determine errors and attain a deep understanding of the underlying structure of the mathematical concept.

Experts rely not only on concepts and procedures when confronted with a mathematical problem. They also have access to metacognition which is the knowledge used by experts in "planning, monitoring, controlling, selecting and evaluating cognitive activities" (Wong, 1989, Herrington, 1990, English, 1992 as cited by English-Halford, 1992; Bernardo, 1997). With this higher order thinking skill, problem solvers are assured of the success of every mathematical strategy they employ.

It is therefore the goal of education to help novices gain expertise in mathematical activities such as problem solving. In the next section, we deal with a few different views of studies conducted on didactics of problem solving.

ISSUES ON TEACHING AND LEARNING PROBLEM SOLVING

Smith, Knudsvig and Walter (1998) advocate a cognitive schema which learners can use to acquire critical thinking strategies. They call it the TCDR for TOPIC-CLASS-DESCRIPTION-RELEVANCE. Thus, when given a learning material, students should ask the following questions:

- What TOPIC I must understand?
- What overall CLASS does this topic belong?
- What is the DESCRIPTION of the topic?
- What is the RELEVANCE of the topic?

(p.3)

These questions help learners interpret, analyze, organize and make sense of the information that are given in the material for better processing of learning. Once this becomes the framework of the learners, they gain strength and clarity of thinking. Several schemes have been offered by mathematics educators for solving word problems.

The most versatile and widely used scheme for problem solving is the one formulated by George Polya (1957). These include

> working simpler problems, restating a problem, decomposing or recombining a problem, drawing figures, making charts or organized lists, exploring related problems, using logical deduction, using successive approximations, using guess-and-check methods, and working backwards. (NCTM, 1989, as cited by Barb and Quinn, 1988, p. 537)

Polya (1957) also developed a framework for problem solving in terms of such general phases as "understanding the problem, divising a plan, carrying out the plan and looking back" (cited by Barb and Quinn, 1997, p. 537). If carried out effectively, then the problem solver becomes successful in handling a problem situation. But the process involved in traversing these steps is quite complex. The learner has to use her prior knowledge, apply acquired mathematical skills, understand the context of the problem situation, and choose the appropriate strategy in solving the problem. This requires formal abstraction, a higher order thinking skill that is available to experts alone. What, then, can be done to help novices gain intellectual power?

By their success in working with simpler problems, novices gain confidence and are motivated to work with more difficult ones. Their analogical thinking can be best harnessed by using very concrete prior experiences. They are able to build their mathematical ideas from simple tasks and are able to acquire mathematical skills. Bernardo (1997) emphasizes the importance of the use of context problems that are familiar to the students which "provides students with a concrete (possibly, real) grounding on the problem, and which allows students to more easily draw from their existing knowledge about similar situations"(p. 11). Hopefully, students become more involved in the difficult task of making learning meaningful.

Mathematics educators recommend the use of mental models to guide learning. These mental models (aids) come in the form of diagrams or drawings used to represent the structure of the concept. The development of strategies and mental modeling fall under the theory of analogies. The effectivity of the analogy lies in a learner's ability to recognize the "correspondence between the structure of the aid and the structure of the concept to be understood" (English-Halford, 1992, p. 121). In this case, the learner is able to map the essence of the model into the essence of the concept, and match or transfer specific conceptual aspects of one domain into another. This cognitive process promotes reflective abstraction. It is unfortunate, though, that certain popular pedagogical practices are counterproductive.

In the process of streamlining the problem solving task, teachers are sometimes tempted to use artificial and fabricated ways of building skills which Blais (1988) refers to as remedial processing. One good example is the prescription of finding key words in a problem which may work for experts, but not necessarily for novices. Some novices use these key words to decide on the algorithm to apply, with complete disregard of the essence of the problem. Key words prompt novices to add when they see the word increase, or subtract when they see the word decrease in a problem. Worse, some apply an arithmetic operation on any two numbers that they see depending upon the key words that they find in the problem. In fact, even their use of formal symbolic expressions in the solutions of the problems may not even communicate the essence of the given problem.

Blais (1988) laments that "[c]onventional instruction permits, allows, and sometimes blatantly encourages algorithmic activity that is separate and isolated from the perception of essence"(p. 627). This may be due to the focus of instruction on the product and not the process of the mathematical activity. In fact, explanations sometimes send the wrong signal that problem solving processes are neat, well organized and easy as the teacher's presentations on the board. Consequently, novices are tempted to resort to rote memorization of the algorithms, rules and formulas presented by the teacher. They do not realize that proficiency in problem solving is best achieved in recognizing the essence of a given problem and the application of the proper problem solving heuristics. Understanding the structural relations in a mathematical problem ushers the learners to reflective abstraction and gives them a sense of direction and feeling of certainty.

Barb and Quinn (1997) advocate the use of multiple methods of problem solving including such intuitively based methods as the guess-and-check method approximation. Problem solvers can use arithmetic computation with figures and charts and logical reasoning, and not necessarily algebraic equations in finding solutions. They believe that this strategy is more meaningful to a learner who is beginning to use some form of reflective abstraction, than rote application of algorithms usually found in textbooks. Teachers who usually look for algebraic solutions should be convinced of the value of developing the students' problem-solving skills and refining their strategies using intuition and logic. It should be noted that the ultimate goal of this instructional method is to help learners build a good knowledge base in solving word problems so they can achieve reflective abstraction in the process.

This belief was expressed by Owen and Sweller (1989) when they challenged the emphasis placed on problem solving and heuristics in the 1980s and pointed out that "superior problem solving performance does not derive from superior heuristics but from domain specific skills" (cited by Puut and Isaacs, 1992, p.215). They claim that the use of general cognitive strategies such as the means-end strategy impose heavy cognitive load and hamper schema acquisition and rule automation. It is because "a means - end tactic involves comparing the initial conditions of a task against the goal set for that task, then searching for a tactic that will transform either the goal or the initial conditions to be a bit more like one another" (Wine & Stockley, 1998, p. 124). This becomes very difficult especially when solving multistep problems. The solver has to analyze and break down the problem to subgoals, successfully transform each initial condition and subgoal into the desired condition, repeat the tactic until the final goal of the problem is achieved. The learner has to see the overall structure of information, concepts, operations, rules, and all other elements that make up the whole schema of the problem. It is preferred that problems be freed of a single goal. When the problem becomes goal-free, solvers are able to work forward from givens of the problem that they are able to generate. According to Wine & Stockley, “each iteration is a self-contained step that uses whichever problem-solving technique is easiest for the student, [in which case] the drain on working memory’s resources is minimized”(1998, p.125). In fact, Sweller (1989) claims that “research shows that freeing problems of singular goals can help students acquire schemas for solving problems “(cited by Wine & Stockley, p. 125). The development of domain-specific skills of learners may facilitate the development of schemas that underlie genuine understanding and meaningful learning.

Another issue that is worth considering is the question of when students should engage in word problems. Word problems are usually treated as application problems since they are given after certain mathematical concepts are introduced, with the aim of using the concepts in solving the problems. On the other hand, word problems may be taught in context, i.e. they may be used to teach a mathematical idea or process. According to Laughbaum (1999) “[t]eaching in context also uses problems or situations, but they are used at the beginning of a math topic for the purpose of helping students understand the mathematics to be taught, or to create a motivating experience of the mathematics to follow” (p.1). Certain groups looked into the effects of application problems to the development of the skills of the learners. One such group called the Cognition and Technology Group of Vanderbilt (CTGV) identified the shortcomings of the application problems and came up with efficient ways of teaching word problems in context. The CTGV has these to say about application problems:

> 1. Instead of bringing real world standards to the work, students seem to treat word problems mechanically and often fail to think about constraints imposed by real-world experiences.
> 2. Single correct answers to application problems lead to misconceptions about the nature of problem solving and inadvertently teaches students for a single answer rather than seek multiple answers.
> 3. The goal of one’s search for a solution is to retrieve previously presented information rather than rely on one’s own intuition. This may limit the development of people’s abilities to think for themselves.
> 4. They explicitly define the problems to be solved rather than help students to learn to generate and pose their own problems. Mathematical thinkers tend to generate their own problems.
> 5. The use of application problems lead to inert knowledge. Inert knowledge is that which is accessed only in a restricted set of contexts even though it is applicable to a wide variety of domain.
> (1997, p. 40)

These application problems are traditionally presented using general problem solving strategies which Polya prescribed or the means-end strategy. While some educators and researchers express the abovementioned concerns, many mathematics educators still adhere to the conventional practices of teaching problem solving.

Lawson (1990), in defense of conventional methods, explained that when done properly, "general problem solving strategies play an important role in learning and transfer" (cited by English-Halford, 1992, p. 120). He described the three different types of general problem-solving strategies to include:

> Task orientation strategies (which) influence the dispositional state of the student and include the broad affective, attitudinal, and attributional expectations held by the student about a particular task. Executive strategies are concerned with the planning and monitoring of cognitive activity, while domain-specific strategies include heuristics such as means-ends analysis and other procedures developed by the problem solver for organizing and transforming knowledge (e.g., constructing a table or drawing a diagram).
> (p. 120).

Lawson insisted that these strategies "have a general sphere of influence on cognitive activity during problem solving and should be seen as distinct from strategies specific to a particular task" (p. 404, 1990, cited by English-Halford, 1992, p.120).

Bernardo (1997) recommends the use of variable problem contexts to promote abstraction. He claims that "[b]y presenting concepts in variable problem contexts, students will come to appreciate the meaning and use of a particular concept or procedure in a variety of contexts"(p.12). Problem solvers cannot possibly recognize problem structure of single problems, thus the need for use of a wide range of diverse problems to facilitate the abstraction of specific concepts and transfer of knowledge to various problem contexts. He believes that a "deeper engagement of the problem information should lead to better conceptual understanding of the problem, and hopefully, result to higher level of abstract thinking about the problems"(p. 13). He proposes teaching strategies that promote analogical transfer. It should be noted that "many theorists argue that specific experiences are represented in memory as cases that are indexed and searched so that they can be applied analogically to new problems that occur"(Kolodner, 1991, Riesbeck and Schank, 1989; Schank, 1990 cited by CTGV, 1997, p.37). It is therefore the task of mathematics educators to determine ways of facilitating analogical transfer among learners.

One such instructional strategy that promotes analogical transfer involves presenting students with a context problem and then asking them to make their own problem using a different context. The effectiveness of this strategy according to Bernardo seems to be due to the

> deeper level of understanding of the problem structure achieved by the problem solver…[as she] explores the problem structure while attempting to create an analog, …[and] as a result of correctly mapping the problem structural information to create a true analog of the original problem. (Bernardo, 1998, p. 7)

Through this problem posing strategy the learners are able to recognize the essence of a problem and construct similar problems with the same essence.

Mathematical problem posing, according to Silver (1994, cited by Ban-Har and Kaur, 1999) "is the generation of new problems or the re-formulating of existing ones "(p. 77). It is recognized as "a valuable process that is motivating, challenging and allows students to exercise their creativity and independent learning skills" (Southwell, 1999; Silver, 1994, Kilpatrick, 1987 as cited by Ban-Har and Kaur, 1999). There are variety of ways to pose problems as a mathematical activity. These include writing questions based on given set of facts, on a given calculation, or on certain information. The benefits of the activity are the same whichever form is used. While results of recent studies give no clear correlation between quality of problem posing responses and problem solving ability (Ban-Har and Kaur, 1999), there are indications that, when performed in the context of analogical problem construction, analogical transfer is facilitated (Bernardo, 1998, p. 7).

There are other ways of facilitating recognition of problem structures, one of which is the use of text editing skills. In this activity, problem solvers are asked to identify missing information from problems or point out information that are irrelevant to the problems. Low and Over (1989) showed the significantly high correlation between students' ability to edit the text of algebraic story problems and their ability to solve these problems; as well as between students' ability to edit the text and categorize problems as being similar or different from each other (cited by Putt and Isaacs, 1992, p. 215). This activity enhances the problem solvers' awareness of their own thinking processes. Such awareness helps learners identify their points of strengths and weaknesses and regulate their own ways of knowing.

Garofalo and Lester (1985) claimed that "most problem solvers do not develop the appropriate metacognitive knowledge that should accompany the execution of computational procedures for doing problems"(cited by Bernardo, 1997, p. 8). Wong (1989) and Herrington(1990) showed otherwise in their studies (cited by English-Halford, 1992). According to Wong (1989), "most students indicated that they were conscious of metacognitive processes and used strategies for monitoring and regulating the processes necessary for problem solving" (cited by English-Halford, 1992, p.118). Herrington (1990) also observed that "upper primary school children had well formed views on the process of learning mathematics and were able to confidently express them"(cited by English-Halford, 1992, p.119). Inspite of these varying opinions, Wong and Bernardo both agree about the need to use guided instruction in the use of metacognitive strategies for problem solving especially among lower ability students. Bernardo (1997) echoed Schoenfeld's suggestion (1987, cited by Bernardo, 1997) that teachers model the metacognitive processes in problem solving when they present solutions to their students. A teacher thinks aloud and exhibits the process of planning, organizing, analyzing and carrying out the solution. The teacher articulates questions, makes mistakes, traces and corrects mistakes, deals with incorrect approach, backtracks, evaluates her progress, and struggles to arrive at the correct solution. This teaching strategy demonstrates the complexity of the process involved in solving problems and the reality that there are many possible ways of arriving at the correct answer.

In the light of all the issues and conflicts on various aspects of problem solving, particularly on developing cognitive strategies among students, and with the assumption that teachers hold wholesome beliefs and attitudes towards mathematics teaching, this paper attempts to offer suggestions on effective ways of fostering critical and analytical thinking through problem solving at different school levels.

FOSTERING CRITICAL AND ANALYTICAL THINKING THROUGH PROBLEM SOLVING

At this point, we all agree that an expert problem solver is a critical and analytical thinker. When a learner gains expertise, she has acquired all the qualities of strong and smart thinking. She becomes insightful, and logical. The expert is also a constructive learner. She participates actively in the learning process and is able to build from her prior knowledge while assimilating and accommodating new knowledge. She appreciates the variety of ways of solving mathematical problems and recognizes a good solution. She is not afraid to use intuition and logic in her solutions. She makes good models of the problems and recognizes the essence and structure of a given problem. She employs a cognitive schema that helps her organize and plan her strategies. Her metacognitive skills help her monitor and evaluate her progress.

Expertise can be attained at an early age. Blais (1988) cites indicators of a schooler's expertise once a teacher expresses doubt in her work. According to Blais,

> [I]f the child does not erase, if she or he refuses to accept the hint from an outside authority and tries to ponder whether the answer is correct, that student is an expert. Being willing and able to think and act independently, she or he will decide what is sensible and reasonable based on informal concepts already acquired (Mills, 1859). A child accustomed to accepting rules and procedures on faith has subordinated his or her own reasoning to outside authority and would have yielded to it once again; the child would have erased. (Blais, 1988, p. 626)

This suggests that teachers should allow their students to experience the joy of working independently by simply guiding and facilitating their learning and by not doing all the thinking and solving for them.

Let us consider the following strategies:

USE OF PROBLEM TYPE SCHEMATA

Valuable instructional suggestions can be gleaned from the Cognitively Guided Instruction project at the University of Wisconsin, Madison. The successful teachers of CGI have a clear idea of the problem schemata type of every word problem that they gave their students. A good example is the following typology made by Carpenter, Fennema, and Franke (1994, cited by Hankes, 1996):

PROBLEM TYPE	ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION
1. JOIN	Connie had 5 marbles. Jim gave her 8 more marbles. How many does Connie have all together?
2. SEPARATE	Connie had 13 marbles. She gave 5 marbles to Jim. How many marbles does she have left?
3. PART-PART-WHOLE	Connie has 5 red marbles and 8 blue marbles. How many marbles does she have?
4. COMPARE	Connie has 13 marbles. Jim has 5 marbles. How many more marbles does Connie have than Jim?
5.MULTIPLICATION	Megan has 5 bags of cookies. There are 3 cookies in each bag. How many cookies does Megan have altogether?
6. MEASUREMENT DIVISION	Megan has 15 cookies. She puts 3 cookies in each bag. How many bags can she fill?
7. PARTITIVE DIVISION	Megan has 15 cookies. She put the cookies into 5 bags with the same number of cookies in each bag. How many cookies are in each bag.

CGI teachers also know the developmental solution strategies that their students employ when solving a problem. An excerpt from the table by Carpenter, Fennema and Hankes (1994), cited by Hankes (1996) is as follows:

CHILDREN'S SOLUTION STRATEGIES	
Direct Modeling Strategies	
Strategy	Description
Matching: Megan has 3 stickers. Randy has 8 stickers. How many more stickers does Randy have than Megan?	A set of 3 objects and a set of 8 objects are matched one-to-one until one set is used up. The answer is the number of objects remaining in the unmatched set.
Trial and Error: Deborah had some books. She went to the library and got 3 more books. Now she has 8 books altogether. How many books did she have to start with?	A set of objects is constructed. A set of 3 objects is added to or removed, and the resulting set is counted. If the final count is 8, then the number of elements in the initial set is the answer. If it is not 8, a different initial set is tried.
Counting Strategies	
Strategy	Description
Counting down: There were 8 seals playing. Three seals swam away. How many seals were still playing?	A backward counting sequence is initiated from 8. The sequence continues for 3 counts. The last number in the counting sequence is the answer.
Counting on to: Chuck had 3 peanuts. Clara gave him some more peanuts. Now Chuck has 8 peanuts. How many peanuts did Clara give to him	A forward counting sequence starts from 3 and continues until 8 is reached. The answer is the number of counting words in the sequence.
Deriving and Fact Recall Strategies	
Strategy	Description
Deriving: Six frogs were sitting on lily pads. Eight more frogs joined them. How many frogs were there then?	The child answers "14" almost immediately and explains, "I know because 6 and 6 is 12 and 2 more is 14."
Fact recall: Eight birds were sitting in a tree. Five flew away. How many are in the tree now?	The child answers "3" immediately and explains, " I know that 8 take away 5 is 3."

With this knowledge, the CGI teachers are able to give their students the wonderful experience of inventing solution strategies to word problems and guide them accordingly. These teachers believe that:

(1) all children know something about mathematics and that part of the teacher's role is to attempt to determine that knowledge base so as to plan instruction;
(2) focusing on problem solving helps reveal children's mathematics knowledge;
(3) encouraging students to invent strategies that make sense to them when solving word problems and sharing such strategies reveals students' thinking as well as facilitates learning.

(Hankes, 1996, p. 454-456)

This approach to teaching primary school mathematics paves the way to the development of higher order thinking skills among pupils and helps them gain expertise at an early age. The confidence that they gain in their own ability to handle word problems motivates them to tackle more complex mathematical problems. This confidence also drives them to be creative in finding solutions to problems.

Another strategy that enhances problem type schemata of students is the use of problem posing. Let us take a look at how several mathematics educators and cognitive scientists employed this strategy among students.

USE OF PROBLEM POSING

We have seen the benefits of acquiring problem type schemata in problem solving activities. Recognition of the structures of the problem leads to the recognition of the essence of the problem This promotes reflective abstraction and consequently critical thinking. Moses, Bjork and Goldenberg (1990) give the following suggestions on how the experiences of middle school students in problem solving can be enriched using problem posing:

1. Have students learn to focus their attention on known, unknown and restrictions of the problem. Then consider the following question: What if different things were known and unknown? What if the restrictions were changed.
2. Begin in comfortable mathematical territory.
3. Encourage students to use ambiguity to create new questions and problems.
4. Teach the idea of domain from the earliest grades, encouraging children to " play the same mathematical game with a different set of pieces.

(Moses, Bjork and Goldenberg, 1990, pp. 83-86)

When given in an atmosphere of collaboration and cooperation, students interact well and participate actively in the activity. This way, they are able to monitor their own misconceptions and make the necessary corrections. If working in a group or with partners, they learn to listen, evaluate and assess the work of others, be open to different ideas and perspectives, and defend their viewpoints in case they disagree on certain points.

Problem posing can also be applied to students using a variety of mathematical tasks that fit their interest and capacity. Various versions of problem posing and problem formulation activities are developed by mathematics educators, educational psychologists and cognitive scientists. An activity developed by Wilson, Fernandez and Hadaway (1993, p. 65) consists of making students list down the attributes of a given mathematical theorem or rule. Then the students are asked to generate new problems if some or all of the given attributes are not true..

The study of Bernardo (1998) used a kind of strategy in problem posing that promotes analogical transfer among high school students. They were given four types of basic probability problems. For each problem type, four analogous problems were developed.   The students were given instructions on the solutions of the problems for each problem type. Students of the experimental group were asked to make their own problems similar to the one they studied. Suggestions on objects and events they can use in the problem were given. Then, the students were asked to solve the problem. The study showed that students who used the problem construction strategy were better at solving the analogous word problems. His study confirms an earlier research which he did in 1994 which showed that "problem solvers retain problem-specific information in problem-type representations because such information affords access to abstract structural information about the problems" (p. 392). His studies clarified the valuable constribution of problem-specific information in the process of acquiring abstract problem-type representation in the learner.

Ban-Har and Kaur (1999) used a problem posing task requiring students to pose problems based on a given set of facts. They established a framework which they called nodal framework for analyzing the correctness and complexity of the problems formulated. They found out that students who were unable to detect contradictions in the information they provided in their problems were consistently unable to solve non-routine problems. There was no clear correlation, though, between the ability to pose good problems and the ability to solve problems.

USE OF INTUITIVE BASED STRATEGIES

Inventiveness in solving problems can be encouraged among novices at the middle school up to the secondary level. It is worthwhile to look at how informal explorations that are initially intuitively based can be used in solving word problems. Allowing students to use invented solution lets all students work at their own levels of abstraction and allows for multiple ways to obtain the same answer, fulfilling a goal of the Standards document (NCTM, 1989, cited by Barb and Quinn).

Consider the examples by Barb and Quinn which allow the use of intuition and modeling in its solutions.

1. Jessica's typing job is worth P8 per page with illustration and P3.50 per page without illustration. She typed 49 pages and earned a total of P293. How many pages of each kind did she type?

The student can be allowed to use the following reasoning:

Since the total amount of the typing job is a whole number, then the total number of pages worth P3.50 has to be even, example 24. Then there should be more P8 pages, say, 27 and there are 22 P3.50 pages. This gives the desired amount of P293.

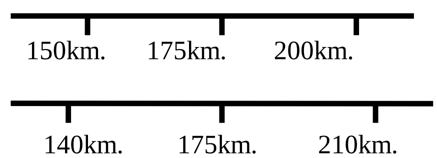
Students may also be allowed to use successive approximations in such problems as the following:

2. An executive leaves home on a business trip traveling 50 kph. One hour later, her husband finds San important briefcase that she left and starts after her at 70 kph. Assuming that no one is stopped for speeding, how long will he take to catch her?

Using the number line, the executive's trip can be illustrated as follows:

The husband's trip one hour later can be illustrated in the numberline as follows:

Since the husband started one hour later, he could not have caught her on the first hour of her trip. It should be some time after the executive's third hour of trip when she has driven between 150 and 200 km. The numberlines on this lap of the trip for both the executive and the husband are as follows:



The husband caught her 175 km. from where they started.

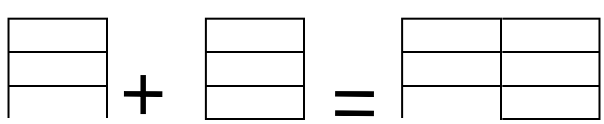
3. A 40% disenfectant solution is to be mixed with a 20% disinfectant solution to obtain 10 liters of a 30% solution. How many liters of the 40% solution and how many liters of the 20% solution should be used?

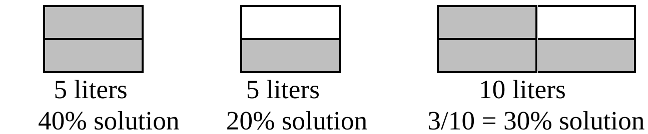
Without using algebraic solution, the students can be led to analyze the given problem by asking such questions as:

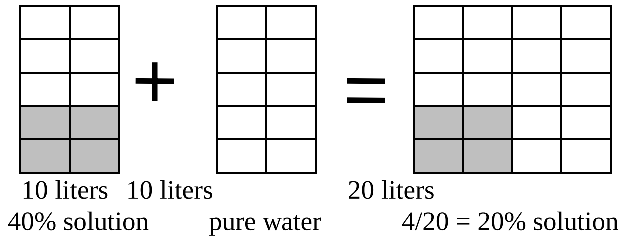
Will equal amount of the 40% solution and the 20% solution work in this situation? Why?

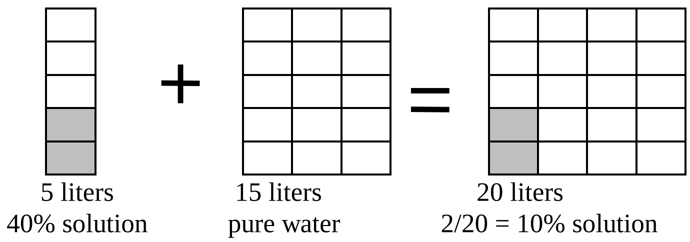
Suppose pure water is to be added to the 40% solution to obtain 20 liters of a 20% soultion, will equal amoung of the solution and water work? Why?

These questions may be accompanied by the following diagrams to help students determine the answers, as presented by Barb and Quinn:









These strategies can be adopted to help students build their skill in problem solving slowly from their physical experience with the real world. Their intuition, logic and visual skills are harnessed in the process. They are able to build from their prior knowledge their problem solving skills in a way that is meaningful to them. In acquiring the intuition for perceiving the essence of a word problem, the foundations for their higher level mathematical skills are likewise built. Besides, their ability to symbolize, represent, generalize and model are enhanced.

USE OF TECHNOLOGY IN MATHEMATICS INSTRUCTION

Mathematics classrooms in many places especially in progressive countries have access to computing technologies and other peripheral devices. Classroom equipment includes scientific and graphic calculators, calculator-based ranger, graph-link, calculator-based laboratory which includes motion detector, microphone, sensors and numerous other gadgets, computers, modems, printers, scanners, word processors, internet browser, electronic mail browser, CD-ROM and other interactive computer-based tools, televisions, video disc players, and all sorts of tools for recording and manipulating information. A lot of research has been conducted on the use of computer technology in education. There is a proliferation of calculator and/or computer based instructional materials in mathematics. Computer-based materials may come in the form of electronic information that can be retrieved from the World Wide Web or as a software designed computer-assisted instruction. There are sites which are devoted to the resources for teaching mathematics. Great attention has been given to computer assisted mathematics instruction, but "little is known about instructional design issues that affect students' learning with technology" (Wine & Stockley, 1998, p107).

One research project that developed high quality materials to support learning is the Jasper project which was conducted by the Cognition & Technology Group at Vanderbilt (CTGV) for 7 years. The Jasper series consists of 12 videodisc-based adventures with video-based analogs, extensions and teaching tips for use in mathematics instruction from the middle school to the higher levels. The eight features of the Jasper adventures are as follows:

> 1. Help students learn mathematics while solving problems in authentic context. The use of mathematics in authenatic contexts supports students' reasoning, problem solving, and communication skills, all standards identified by the NCTM (1989).
> 2. Provide a context that helps students integrate concepts in mathematics as well as mathematical knowledge with knowledge of other subjects.
> 3. Take advantage of the power of video and interactive technologies. Video allows a more veridical representation of events than text. It is dynamic, visual, and spatial, and students can more easily form rich mental models of the problem situations (e.g., Johnson-Laird, 1985; Mc.Namara, Miller & Bransford, 1991; Sharp et al., 1995).
> 4. Support Inquiry. The adventures are designed to help students understand the kinds of problems that can be solved through mathematical inquiry. The adventures also include embedded teaching that often takes the form of modeling by experts (Brown, Collins, & Duguid, 1989). Modeling can also provide coaching and scaffolding for students as they develop their own skills (e.g., Vygotsky, 1978, 1986).
> 5. Students must generate as well as solve problems. The adventures end with challenges that specify a general goal for the students. Nevertheless, in order to solve the challenges, students must identify a number of subproblems and generate subgoals of their own.
> 6. Provide opportunities for collaboration over an extended period of time. As students work together over multiple class periods (from several days to several weeks) to solve a challenge, they have repeated opportunities to communicate about mathematics, share their ideas about problem solving, and receive feedback that helps them refine their thinking.
> 7. Afford students the opportunity to develop a deep understanding of mathematical concepts. Each videodisc adventure also includes video-based analog and extension problems. These problems help students engage in what-if thinking by revisiting the original adventures from new points of view.
> 8. Provide positive role models. A goal of the Jasper series is to provide positive role models for students from all backgrounds.
> (CGTV, 1997, pp.3-8)

Several types of studies were conducted to see how the use of the Jasper series affected learning and transfer of learning. The formative assessment conducted on problem-based instruction using the Jasper series showed positive results in terms of increase of students' learning and problem-solving performance. Meaningful learning was evident when the students designed projects that are tailored to the local community. One such project is the creation of a business plan for a fun fair to be held at their school. Studies showed the importance of the experiences that they gained as they engaged in a collaborative work on these projects.

The Jasper series is one component of a bigger project which explores ways modern technologies can be used in mathematics instruction. This is called the Schools for Thought (SFT) project. The goal of this project is to restructure curriculum, instruction, assessment, professional development, and community participation in ways that help students develop the competencies and confidence necessary for success in the 21st century (Williams, Burgess, Bray, Bransford, Goldman, & CTGV, 1998, p.97). SFT also utilizes technology to provide multiple resources for feedback and revision. This is called the SMART Challenge series. SFT classrooms showed that technologies do support student understanding and provides resources and scaffolds that promote deep understanding and enhance learning. Besides, the value of computer-mediated communication to establish collaboration among students, teachers and the bigger academic community was evident in the professional interactions that they had.

A reform effort that is taking place in Union City, New Jersey is the Project Explore, Union City Online: An Architecture for Networking and Reform. This project has helped to develop a technical infrastructure that delivers high speed Internet connectivity to the 11 schools in the district ...[and] was charged with the development of an effective and sustainable human infrastructure (Honey, Carrigg, & Hawkins, 1998, p.122). Participating schools collaborated with the Math Forum project at Swarthmore College which designed internet-based materials for mathematics classes. Interactive communication regarding concerns about mathematics instruction is available in the project's Web site entitled Linking Math Proficiencies to Internet Resources. Union City is also a site for New Jersey's Systemic Initiative (SSI) which makes it a model of reform in math, science, and technology. As such it participates in various projects like the Woodrow Wilson Scholars-LEgo Robotics Program which "incorporates cooperative learning, mathematics, science, and technology"(p. 134).

A research project on project-based learning is conducted by EduTech Institute at Georgia Institute of Technology. The project focused on making design problems effective learning opportunities by "reducing costs with minimal time and effort on the part of the teacher, improving learning benefits, and creating practical project based learning opportunities for large numbers of students" (Guzdial, 1998, pp. 48-49). To ensure effective learning, researchers of EduTech require that "students be given opportunities to reflect on their learning by focusing goals on knowledge building; and that enough support be provided to them"(p.48). One such support for student design activities is a Web-based collaboration tool called the WebSMILE (Web-Scaffolded Multi-user Integrated Learning Environment). This is based on the program of Roland Hübscher where a flowchart developed by Sadhana Puntembakar provides students with the things they need to do in the problem-solving process. Similarly, "GPCreditor (Goal-Plan-Code editor), a learning environmentfor high school students studying design through Pascal programming, also scaffolds the planning process and integrates it with the problem-solving process"(p.54). It claims that high school students demonstrate expertise in programming after the use of this design support and are able to retain the skill even when programming in a traditional environment.

Researchers attest to the success of the projects on technology-based mathematical instruction. Educators recognize the partnership that teachers and students can establish with computing technologies for effective mathematics teaching and learning. The best use of computing and multi-media technologies is in the context of support for mathematics instruction. This has to go with pedagogical principles that are deeply rooted in sound philosophies of knowledge and education. At this point, let us look into some college mathematics programs that used computing and multi-media technologies.

SOME TECHNOLOGY BASED COLLEGE MATHEMATICS PROGRAMS

The Ohio State University, Mathematics, Science, and Technology Education department of the School of Teaching and Learning conducted a Remedial Mathematics Pilot Program in 1996. It is basically a two quarter course in algebra with emphasis on problem solving, reasoning, communication, connections by using multiple representations, manipulatives, graphing calculators and any tool that enables students to learn the concepts for understanding. Mathematical concepts were oftentimes embedded in problem situations. Graphing calculators were made available for students to check out every class period. The teacher had an overhead graphing calculator to aid in discussion and presentation. Students worked with algebra tiles for combining like terms, multiplication of binomials and factoring until they are comfortable with the concepts. Topics include: linear equations and inequalities, graphing, polynomials, perimeter and area, systems of equations, radical expressions, quadratics, and rational expressions. A one year progress report of the success of the program was prepared by the faculty advisor, Dr. Patricia Brosnan and the instructor, Ms. Denise Forrest. Some of the findings were:

1. Students' attitudes towards mathematics improved.
2. Students became more confident in their knowledge about and self efficacy towards mathematics.
3. Students not only improved their problem solving and reasoning abilities, but enjoyed and oftentimes preferred working on word problems.
4. Because communication was an integral part of the course, students' ability to articulate their mathematics knowledge improved dramatically.
5. Cooperative learning and small group activities were important for student problem-solving success.
6. Valuing student thinking and teaching by not telling provides the student the opportunity to really understand mathematics.
7. Multiple representations appealed to more learning styles, thus reached more students for understanding.
8. Establishing a caring environment makes an important intangible difference.

(Brosnan & Forrest, 1996 pp. 8 - 10).

Another program that adheres fully to a problem-based and technology-based curriculum in mathematics is the program designed by Mr. Edward Laughbaum of the Ohio State University Technology College Short Course Program. He designed materials which can be used for teaching in context at the developmental level. Some of the features of his materials which were recommended in the AMATYC Standards - Crossroads in Mathematics, and developed as a book are as follows:

1. Technology is integrated throughout to enhance the learning and teaching of mathematical concepts and to provide options for performing mathematical algorithms.
2. Mathematical concepts are introduced in the context of real-world situations.
3. There are guided discovery exercises.
4. There is increased emphasis on the use of function as a central theme. Learning spirals from an intuitive idea of function to formal treatment of the concept requiring higher level thinking.
5. Numeric, graphic and algebraic methods of representing functions are utilized.
6. Students are encouraged to explore on their own and use various methods for solving problems.
7. Activities include projects like the extended laboratory projects on modeling; exercises with varying levels of difficulty, and which include open ended questions, concept questions, writing questions, and exploration problems.
8. Students are encouraged to use both the numerical and/or graphical checking of problems.
9. Group work is encouraged.

Laughbaum (2000)

Topics include numbers, functions and their graphical representations, analysis of linear, quadratic, absolute value and square root functions, operations on polynomial functions, factoring, equations and inequalities containing the linear expression and the absolute value expression, formulas, direct variation, exponential function, equations and inequalities with exponential expressions, rational functions, fundamental properties and operations of rational functions, solving equations and inequalities with rational expressions, square root function, irrational expressions and their operations, fractional exponents, quadratic function, solving quadratic equations, geometry, trigonometry, systems of equations and inequalities, and logarithmic function. There is mathematical modeling of linear functions, exponential functions, square root functions, quadratic functions, trigonometric functions, logarithmic functions and systems of equations.

Professors Bert Waits and Frank Demana, founders of the Technology College Short Course Program of the Ohio State University prepared a set of activities using the computer algebra calculator and other calculator based gadgets. Their compilation of activities is entitled Mathematically Modeling Science **M^2 S**, Enhancing the Teaching and Learning of Mathematics & Physics with Hand-Held Technology. It consists of activities that integrate mathematics and the physical sciences aimed at enhancing comprehension of mathematical and scientific concepts. The activities use various Texas Instruments equipment.

The special feature of the abovementioned programs is the use of problem solving to build on the mathematics concepts and motivate the learners to engage in the mathematical task. They share the belief that meaningful learning in mathematics takes place in a problem-based curriculum. It is however important to use technology to help in the learning and teaching processes. Consequently, the activities use graphing calculators heavily.

From the various issues and ways of negotiating the issues that have been discussed, an attempt is made to draw conclusions regarding what a good technology-based curriculum should be for college algebra.

A TECHNOLOGY BASED CURRICULUM IN COLLEGE ALGEBRA

Problem solving is seen as the manipulation of an internal mental model of the external world. In the process of finding the solution, “we solve the problem in the internal representation and then project its solution into the thing being represented”(Hunt, 1994, p. 218). The solution is brought about by the manipulation of the representation by a human and/or an electronic thinking device. Learners construct a mental model of the situation in their memory. The learner's symbolic represemtation and manipulation is a limiting feature of human problem solving, though. Newell and Simon (1961,1972) proposed that computer programs can be gleaned as models of human thought and then offered the following insights:

> 1. A theory of the process of problem solving can be expressed as a program, that is a set of rules for manipulating symbols. Indeed, if a theory is proposed that cannot be so expressed, that theory is unacceptably vague.
> 2. The development of an ideal problem-solving program in some field of endeavor is a goal in Artificial Intelligence.
> 3. A problem- solving program that, in some nontrivial sense, behaves like a human being, is a descriptive theory of human problem solving.
> (cited by Hunt, p. 218)

It becomes clear that technology is an efficient partner of humans in problem solving. Since the success in problem solving is determined by the learner's capacity to represent an external situation into symbols and manipulate these representations, then they have to make use of some cognitive tools in the process.

Cognitive scientists believe that learners usually memorize a variety of schemata in order to cope with the problem solving task. This is where the partnership between technology and humans becomes essential. Hunt (1994) believes that " as along as the students have pattern-recognition rules that tells them when to apply which of their many contradictory schemata" then the problem solving skill has been acquired. They need not have an orderly progression of schemata like what computer programs have. More important than the procedures and algorithms is the meaningful understanding of the concepts applied in problem solving. That way, "[s]chemata problem solving works because it moves the computational burden from immediate memory, where the human problem solver is weak, to long-term memory, where the problem solver is strong" (Hunt, 1994, p.231). Since problem solving requires higher level cognitive skill, any mathematics course becomes meaningful if embdded with problem solving tasks.

This paper adheres to a problem-based curriculum in college algebra which provides opportunities for problem solving and mathematical investigations founded on a constructivist theory of learning. It advocates activities that foster critical and analytical thinking in an environment of human support from the teacher, and fellow students, as well as technological support that are available in school.

Problem based curriculum promotes learning with understanding in the first course of college algebra. Word problems can be used to build the mathematical concepts to be discussed. At other times, word problems can also be used as applications of concepts that were introduced. If formulated in the real world context, these word problems can be motivating since the learners see how different mathematical concepts can be used in daily transactions. Modern technology lightens the algorithmic burden of the mathematical task, and eases the cognitive load by helping the learner acquire a problem schema. Even at the college level, computing and information technology are important aids to learning.

Effort should be exerted though to find and/or design good instructional materials for the desired learning outcomes. These materials should take into consideration the technology that are available to the students, be it the modern or the traditional technology of pencil, paper, chalk and board. Well designed activities help build a mathematical community within the classroom made up of the teacher and the students that support each other. If communications technology is available in class, this mathematical community can extend to other mathematics teachers in school or outside, other students engaged in mathematical activities, mathematicians and the community of people from whom information can be gathered on some mathematical projects. Students can be allowed to engage in community-wide converstations about certain mathematical tasks or projects. On the other hand, teachers can build a network among other professionals to design a curriculum for their algebra students. This way, the community can contribute their insights into the curriculum which they think are powerful and relevant to the demands of society.

This paper does not advocate specific topics and activities. Instead, teachers are encouraged to look into the projects specifically funded by such agencies as the National Science Foundation or research findings from experiences in mathematics instruction such as that of the Ohio State University. The classroom teacher is the ultimate curriculum designer. She is at the center of all the decisions about revisions and implementation of the curriculum She is the reliable resource of information about pedagogy, administrative decisions on classroom needs, techonological designs of instructional materials, school reform and restructuring.

The framework of the techno-mathematics curriculum design for algebra at the collegiate level thus recommended is as follows:

FRAMEWORK OF THE TECHNO-MATHEMATICS CURRICULUM DESIGN FOR COLLEGE ALGEBRA		
Learning Theory	Features of Problem-based College Algebra	Pedagogical Implications
Building on the learner's prior knowledge	Probing questions to map learner's concept Single goal problem types Real world problems	The teacher facilitates learning by linking prior knowledge to the new mathematical concepts through inquiry. She selects tasks that incorporate previously learned concepts and enable new mathematical understanding to grow. The teacher encourages the students to communicate their ideas.
Active construction of learning Multiple representation Situated learning Strategic thinking	Goal free word problems Building mathematical skills Mathematical explorations and investigation Generalization of patterns and constraints A body of formal symbols Study of structures Study of functions and relations Mathematical modeling	The teacher lays down the objectives and expected outcomes of the mathematical tasks. She selects activities from various sources that facilitate understanding and meaningful learning. She supports the student's investigative processes. She allows students to use intuition and logic aside from their algebraic skills in solving problems. She also encourages learners to use various strategies including guessing and estimation with the goal of helping them gain expertise in the process. She finds ways to make all forms of algebraic reasoning available to students and help them gain meaningful learning. She gives them opportunities to reflect on their thinking and reorganize their learning.
Mathematical Community of Learners Collaboration Social context Negotiated meaning Distributed expertise	Cooperative learning Environment of mutual support Established norms Sustained focus Student accountability for learning	The teacher establishes an environment that is conducive to collaboration and mutual support, as well as class norms that encourage learning with understanding. The teacher is able to encourage student autonomy and accountability for their learning. They articulate their mathematical thinking, views and insights and critique each other's work. At the same time they demonstrate respect for each other's capabilities and help each other gain the desired expertise.
Cognitive tools including human and technological support	Traditional and/or modern technology Problem schema Analogic transfer Feedback and assessment	The teacher should gain the skill to use the appropriate technology that is available in school. This technology should be used to help students gain problem schema in problem solving. The teacher should be able to help students use the available technology. The teacher presents concepts in variable problem context to promote abstraction. She uses various ways to promote analogic transfer. She focuses on the learning processes and unique thinking of each learner and provides feedback efficiently. She ensures that all students learn mathematics with understanding.

One powerful tool used in mathematics classes is the graphing calculator. Certain concepts can easily be built empirically by encoding data in the calculator and observing the resulting mathematical model. Nevertheless, Waits and Demana (1998a) believe that paper-and-pencil computation can help learners validate technology. In the absence of modern technology and with the use of the tradional technology of chalk, board, pencil and paper, the desired curriculum for college algebra can still be attained.

CONCLUSION

Mathematics educators recognize the need to develop critical and analytical thinking through problem solving. This paper presented the various issues about problem solving that have been raised in the last two decades. Upon analyzing all arguments, this paper embraced the belief that establishing a cognitive schema in problem solving will lessen the heavy cognitive load of the problem solving task. Then this paper suggested ways to establish problem type schema among the students at different levels. In teaching problem solving at the elementary level, certain practices of the Cognitively Guided Instruction project may be employed. This includes awareness of problem schemata typology that teachers employ in class and knowledge of developmental solution strategies in assessing learner's solutions to problems. Another activity that enhances schema recognition is the problem posing task. The problem posing tasks are varied and have been proven to promote analogic transfer among the learners.

The paper pointed out the importance of gaining ample knowledge in problem solving for critical thinking to take place in that particular setting. The paper showed how alternative solutions to problems can be encouraged using logic, reasoning, approximation, estimation and visual representations. These alternative solutions allow novices to harness their intuition to gain the expertise needed in problem solving. This way, they can take active part in building knowledge and gain expertise in the process.

The role of technology as a cognitive tool and partner in mathematics instruction was recognized. Some research-based projects on the use of technology in mathematics instructions were cited. Some programs on problem-based mathematics courses at the Ohio State University were also cited. These programs affirmed the benefits of the use of modern technology in promoting meaningful learning.

A framework of a problem based curriculum for college algebra was recommended. While it is believed that computing and information technologies facilitate learning in a mathematics course, the use of modern technology is not vital in the proposed curriculum. Instead, it emphasizes the learning theories and pedagogical aspects of the curriculum, based on the constructivist theory of active building of knowledge that promotes learning with understanding.

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