Using Technology to Implement a Constructivist Approach to Calculus and Abstract Algebra

John Selden and Annie Selden
Department of Mathematics, Tennessee Technological University, Cookeville, TN 38505

This is a report on two yearlong experimental courses and an analysis of the technology needed. The research aim of the project is to gather information useful in the development of future courses that produce students who can use their knowledge in flexible and creative ways to solve nonroutine problems. In a study of first calculus students done in Spring 1988, we concluded that average students coming from small, well-taught traditional classes cannot work nonroutine problems [2].

In a traditional calculus course many students rely heavily on remembering sample solutions. This produces inflexible students, unable to understand problems and applications they have not previously seen. In addition, such students are often deficient in conceptual understanding, remember the material poorly, and tend to find calculus boring.

The courses described here are meant to be similar to traditional ones in content, credit, and manpower requirements, while allowing for a constructivist approach to their teaching. By a constructivist approach we mean that the teacher observes and uses a student's conceptual base in guiding his construction of further concepts and skills [5]. The opportunity to understand a student's thinking arises naturally in one-on-one tutoring but is uncommon in traditional lecture courses.

DESCRIPTION OF THE COURSES. Although there are many possible ways of implementing constructivist ideas, most would involve considerable interaction between the teacher and the students. In our approach, students present their own solutions to problems in class. It is best if the problems are *cognitively nontrivial*, i.e., if the students have not previously seen solutions to essentially similar problems. For a discussion of problem solving in this sense, see [1]. Since adding a significant amount of this sort of work to a traditional lecture course could more than double its length, we decided to remove many of the explanations, sample solutions to problems, etc., and instead, converted this material into problems for the students to solve. Students must explain and justify their solutions when presenting them to the class and we provide criticism.

In both first calculus and graduate algebra our goal is not only that students should learn the usual course content but also that they should be able to use it effectively in new contexts. In calculus this means students should be able to solve concrete, but cognitively nontrivial problems and provide arguments for the validity of their solutions. In graduate algebra, students should be able to solve problems and prove theorems not covered in the course. Often these goals are not met by the traditional lecture method. Lecturing can work well for more sophisticated students in graduate algebra, but more naive students often obtain only superficial knowledge which they are unable to apply. For an analysis of misconceptions and errors made by undergraduate abstract algebra students when taught in this way, see [3].

We give very few lectures; most arise naturally from student questions or our critique of student work. Classroom time is spent in student presentations and our criticisms thereof. Students are expected to solve problems before class, not extemporaneously. Only rarely do we lead the class or an individual student through a problem as this tends to produce an unjustified illusion of success on the part of both teacher and student. Often we rewrite a student's presentation in a clearer way, explaining why our exposition is better. The emphasis here is on obtaining a presentation clear enough for anyone to judge its correctness in the absence of sample solutions with which to compare it.

CLASS ORGANIZATION. In order that all students be sufficiently challenged, we first call on those students whom we think likely to have solved a particular problem with the greatest effort. We also call on students who have not yet presented much, especially when we consider a problem relatively easy. For each student, we record those problems which he has solved correctly but make no record of his incorrect attempts. In graduate algebra, there are no tests, and grades are set according to the difficulty of the problems a student can work. In calculus, students are divided into groups of 5 or 6 and encouraged to work together. Points are assigned to the whole group when any member presents a correct solution. This not only enables us to handle 30 students but also encourages students to articulate and evaluate their ideas before presenting them in class. In calculus, we also have tests, but each test has fewer problems than normal because we never ask problems which have occurred before and each test covers the entire course to date. On tests students are encouraged to use their notes, as well as computational devices. Presently each student has an HP-28S calculator; we hope to add 30 PCs next year.

THE TECHNOLOGY NEEDED. Both students and teachers need technological support for such courses. Students attempting to solve a problem without a sample to mimic will often want to make computational experiments. Certainly our calculus students use their HP-28S calculators in this way and they could use more computing power. In addition, since this sort of teaching is slow, it is useful to streamline the syllabus. There are several topics in calculus which can be covered less intensively provided the computing tools used are always available to the students. For this reason inexpensive technology is especially interesting to us.

While we do not now use computational technology for our algebra course we think it would be useful. It should not be difficult to adapt even the HP-28S to operate on a large variety of algebraic structures.

An altogether different technological requirement of students in such courses involves the notes. Notes are essential because standard textbooks contain sample solutions and cannot be continuously adjusted to the progress of the students. We envision the notes as a problem solving tool, more like a computer manual than a textbook. For efficient use they should be concise and cross-indexed. In addition, they should be expandable so students can add sample solutions, proofs, comments, etc. from classwork without destroying the organization. We now write notes by hand on only one side of each page and number pages decimally, e.g., pages 7, 7.1, 7.11, 7.2, 8. We are careful that items such as theorems, examples, etc. not contain page breaks. We number items by pages, e.g., 7E5 is an example (hence, the E) which is the 5th item on page 7.

The details of the organization of the notes are important as these can effect the psychological accessibility of the information. This can be illustrated with a classroom experience. On the first try, none of our students could work: 37E2 *Show that* $f(x) = sin\ x$ *is continuous at all numbers, a.* On the other hand, adding / 22T2, 37T1 (refering to a list of trig identities and an alternate form of continuity) produced five or six volunteers, one of whom worked the problem correctly. (Here E refers to "example" and T refers to "theorem".) Well organized notes can help to gradually free students from the need for hints.

Ideally the notes should also be available to students in a hypertext version, i.e., presented on a computer in a way so that relevant definitions, theorems, etc. can be called up quickly when needed to help solve a problem [4].

The need to produce notes which correspond to the sophistication of the students and the style of the teacher calls for a technological aid, as the production of notes by hand is very labor intensive. What is required here might be called a *dynamic book*, i.e., a large master set of notes in TEX on disks from which the teacher could select on a weekly or daily basis. Such a dynamic book should also contain (1) a specialized editor to facilitate the teacher's selection and supplementation of the notes, (2) adequate cross-referencing to encourage the use of the notes as a problem solving tool, and (3) provision for insertion of student solutions as they are developed.

INITIAL STUDENT REACTION. We recently distributed a questionnaire to gauge initial student reaction to the calculus course. We found students to be generally pleased with the course. Most favored working in small groups. Some would prefer more explicit instruction on the use of the HP-28S. A few are computer oriented and know as much about the calculator as we do.

We have a graduate assistant assigned to the course whose job it is to observe the class and make a written transcript of what occurs. Not surprisingly, he has observed that some students take rather complete notes, whereas others take hardly any. Eventually we want to examine the relationship between student note-taking and grades, but it seems premature now as the course has just begun. Another observation the assistant made was that despite student presentations being central to the course, the teacher still spends half of the time talking. This includes time spent in course organization and explanations of calculator use, as well as actual critiques of student work and an occasional mini-lecture. This suggests that active student participation in the form of student questions and boardwork in a traditional lecture course must be rather small. It also suggests that the course is not such a radical departure from the traditional one as might be supposed.

REFERENCES

1. Schoenfeld, Alan H., *Mathematical Problem Solving*, Academic Press, Orlando, 1985.
2. Selden, John, Alice Mason, and Annie Selden, "Can Average Calculus Students Solve Nonroutine Problems?", preprint.
3. Selden, Annie, and John Selden, "Errors and Misconceptions in College Level Theorem Proving," *Proceedings of the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics*, Cornell University (July 1987), 457-470. Available through ERIC.
4. "Special Issue: Hypertext," *Communications of the ACM*, Vol. 31, No. 7 (July 1988).
5. von Glasersfeld, Ernst, "Learning as a Constructive Activity," *Problems of Representation in the Teaching and Learning of Mathematics*, Lawrence Erlbaum Associates (1987), 3-17.