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THE IMPACT OF COMPUTER PROGRAMMING ON TEACHING COLLEGIATE MATHEMATICS.

The growth and development of computer technology during the last four-five decades has stimulated new approaches in teaching Collegiate Mathematics.

The purpose of this paper is to present a teaching model which deals with various classes of solvable uniform mathematical problems from Collegiate math courses.

Computer programming processes involve techniques and approaches being convenient tools in developing abilities for problem solving, logical reasoning.Investigating the process of solving mathematical problems by college students we found that more than half of all math students experience difficulties in solving math problems.To estimate the degree of difficulty we used evaluation functions,that is,how long one is wandering before the right direction is found.Our conjecture is:

THE INTELLECTUAL ACTIVITIES OF SOME MATHEMATICS STUDENTS CAN BE PARTLY GUIDED DURING THE PROCESS OF SOLVING MATHEMATICAL PROBLEMS.

Our teaching model reduces the difficulties in cases when a problem belongs to a class of uniform problems.

Creating and using a problem-solving model for a set of uniform mathematical problems from the undergraduate math curricula is perhaps the most fascinating aspect in mathematics teaching process.It should be realized that the difficulties which one meets in the process of mathematical thinking during a problem-solving situation can be critical and it is obvious that the problem on hand will never be solved without an outside impulse.It is our conviction that the most efficient impulse is an algorithmic prcedure created and formulated for the class of problems being considered at a given time.

We define an algorithmic procedure (AP) as a

SEQUENCE OF STEPS LEADING TO THE SOLUTION FOR A CERTAIN SET OF UNIFORM PROBLEMS WHERE DETAILS OF SOME STEPS CANNOT BE DETERMINED BEFOREHAND.

The knowledge of an AP is not a garantee to find the solution of a problem from a given class,it provides a proper path,a general direction subdivided into a number of steps.To perform the actions inside each step one should have the appropriate abilities and skills in order to support the mathematical activities until the problem is solved.That clarifies the concept of an AP: it is only a frame to be used in critical situations,it is free of a detailed description of steps which cannot be subdivided any more.

There are problems arising during the process of forming AP.We will specify just a few of them:

1.HOW SHOULD BE DESIGNED THE CRITERION OF DECOMPOSITION A PROCESS INTO SIMPLER SUB-PROCESSES?

2.WHAT SHOULD BE ACCEPTED AS THE FIRST-LEVEL ELEMENT (WHICH CANNOT BE DECOMPOSED ANY MORE) IN THE PROCESS OF SOLVING MATHEMATICAL PROBLEMS?

3.WHAT ARE THE MATHEMATICAL ABILITIES AND SKILLS OF MATHEMATICS STUDENTS WHICH WILL SATISFY OUR REQUIREMENTS?

4.IS THE LIMITED TIME PREDETERMINED FOR A CERTAIN TOPIC COMPARABLE WITH THE AMOUNT OF KNOWLEDGE TO BE ACCOMPLISHED BY A MATHEMATICS STUDENT?

Our findings show that it does not exist one and only one way to decompose an entire process into less complicated sub-processes. In many cases it is impossible to predict and to determine in advance the size and the content of a certain sub-process. Moreover,a certain subprocess can be considered as a very simple one from person's A point of view and very complicated from preson's B point of view.

There is an additional problem to be cosidered:WHAT ARE THE DIFFERENCES ON SPECIFYING A PROBLEM-SOLVING PROCEDURE BY A HUMAN AND BY A COMPUTER?

We are considering only one aspect in copmarisons:THE PROBLEM-SOLVING PROCEDURE.It is appropriate to observe only a few of them:

1.In general a human needs only the major steps of the procedure, having always enough room for creativity,while a computer program must be designed in such away that all first-element steps must be indicated.

2.A human has a common sense,is constantly in a thinking "mode", acts in a reasonable way,while the computer cannot exercise judgement unless it has been provided with explicit directions for making a decision.

3.Specifications can be reduced to a general description when they are designed for a human use,while the computer accepts procedures being described in a certain language related to an exacting technique requiring attention to details.

Our approach is based on the logic of mathematics itself and on the accessibility (in terms of common sense) of the structural, logical,reasonable construction of the AP.That approach has a tendency to raise the opportunities of the cogitative activity on a higher level,namely to establish favorable conditions for independent creativity.The training to create and use AP provides a tool to solve mathematical problems,to develop one's critical type of thinking and abilities for insight.

Consider now four classes of problems from College mathematics.

1.Find an inverse function $f^{-1}(x)$ for a given monotonic function f(x) determined in an interval I.The corresponding AP can be formulated as it follows:

Step 1. Express x in terms of y: $x=f^{-1}(y)$

Step 2. Exchange x and y: $y=f^{-1}(x)$i.

It is obvious that step 2 is precise and clearly determined. Meanwhile, step 1 describes only tha general action,that is,the requirement to express x in terms of y. In some cases the inversion is very difficult or even impossible (for example,when f(x) is a polynomial of degree five or more).

2.A rational function of trigonometric functions R(sinx,cosx) is to be integrated.The task is to examine the given function in order to establish an appropriate method of integration and to describe as much as possible the corresponding AP. The analysis of that problem results in four available types of substitution,each of which will convert the given trigonometric function into rational function with respect to t: a) cosx = t, b) sinx = t, c) tanx = t, d)$\tan\frac{x}{2}=t$.

To be able to make the right choise we will investigate different possibilities for the expression R(sinx,cosx)

2.1 Let the function R(sinx,cosx) be odd with respect to sinx, that is, R(-sinx,cosx) = -R(sinx,cosx). In that case the given function can be represented as $R(sinx,cosx)=R_0(\sin^2x,cosx)sinx$ The substitution cosx = t transforms the function R(sinx,cosx) into a rational function of t.

Example. Integrate:

$$\int \frac{\sin^3x}{cosx-3}\,dx.$$

The integrand is a rational function with respect to sinx and cosx and an odd function with respect to sinx,therefore the substitution cosx = t converts the given integrand into a rational function of t:

$\sin^3x=\sin^2x\ sinx$. If cosx = t,then -sinx dx = dt, $\sin^2x=1-\cos^2x$. Now the given integral is transformed to the form:

$$\int \frac{\sin^3x}{cosx-3}\,dx=\int \frac{t^2-1}{t-3}\,dt.$$

It is obvious that the same substitution can be applied in case when $R(sinx,cosx)=R_1(sinx)cosx$.

2.2 Let the function R(sinx,cosx) be odd with respect to cosx, that is, R(sinx,-cosx) = -R(sinx,cosx). In that case the given function can be represented as $R(sinx,cosx)=R_0(sinx,cos^2x)cosx$. Referring to the case 2.1 it is easy to see that the substitution sinx = t can be applied in order to convert the given rational function with respect to sinx and cosx into a rational function with respect to t.

2.3 Let the function R(sinx,cosx) be even with respect to both sinx and cosx,that is, R(-sinx,-cosx) = R(sinx,cosx).We will prove now that the substitution tanx = t converts the given function into a rational function with respect to t.Indeed,if tanx = t,then

sinx = t cosx,and $\sec^2x\ dx=dt$.Knowing that

$\sin^{2k} x = (1-\cos^2 x)^k$, and that $\cos^2 x = \dfrac{1}{1+\tan^2 x} = \dfrac{1}{1+t^2}$

we conclude that all expressions involved in the integrand and dx are rational functions of t.

2.4 If conditions in cases 2.1-2.3 are not holding then it is appropriate to apply the universal substitution $\tan\frac{x}{2} = t$.

3. Consider the general form of a linear second order homogeneous differential equation with constant coefficients: $a_0\, y'' + a_1\, y' + a_2\, y = 0$

The AP to solve that differential equation consists of the following steps:
Step 1.Seek the solution in the form: $y = e^{kx}$.
Step 2.Find the first and the second derivatives for y and set up the characteristic equation: $a_0\, k^2 + a_1\, k + a_2 = 0$.
Step 3.Compute the discriminant of the characteristic equation:

$D = a_1{}^2 - 4\, a_0\, a_2$.
Step 4.If D $>$ 0 then k_1 and k_2 are real and distinct numbers.The solutions are:

$y_1 = e^{k_1 x}$, $y_2 = e^{k_2 x}$
The general solution in that case is:

$Y = C_1 e^{k_1 x} + C_2 e^{k_2 x}$.
Step 5.If D $=$ 0 then $k_1 = k_2$ are repeated real roots.
The solutions are:

$y_1 = e^{kx}$, $y_2 = x\, e^{kx}$.
The general solution in that case is:

$Y = C_1 e^{kx} + C_2\, x\, e^{kx}$.
Step 6.If D $<$ 0 then a + bi(a and b are real numbers) is a complex root of the characteristic equation.Since the original equation has real coefficients the conjugate complex root is: a - bi.
The solutions are:

$y_1 = e^{ax}\, cos(bx)$ and $y_2 = e^{ax}\, sin(bx)$.
The general solution in that case is:

$Y = e^{ax}\, [C_1\, cos(bx) + C_2\, sin(bx)]$.

4.The main part of the solution of a system of linear equations using Gauss-Jordan method is devoted to the conversion of the given matrix (augmented) into a special form.Following is the AP to perform this part:

Step 1.Find the column j that contains non-zero entries.
Step 2.Find the row i which contains a non-zero entry in the intersection with column j.Let that entry be a_{ij}
Step 3.Divide row i by a_{ij}.
Step 4.Add suitable multiples of row i to all rows 1,2,...,i-1,i+1,...,n,so that all entries in column j excluding the i row become zeros.
Step 5.Consider a new matrix not containing row i and column j, and go to Step 1.Continue in this way until it is impossible to follow the steps futher.

A considerable part of mathematical applications are related to verbal problems which arise in different areas of human activities. The history and nature of mathematics itself is a chain of findings and discoveries related to various scientific verbal problems.

The process of solving a word problem at any level of mathematical education is creative and not predictable.We cannot formulate in advance a prescribed set of steps ,that is,an algorithm which will solve any given word problem.On the other hand it is possible to illustrate some patterns of solving word problems and to emphasize the common parts showing the general approach of this type of mathematical thinking.The AP reflecting that type of problems is:

Step 1.Read careful the given verbal problem (more than once).
Step 2.Write down the given numerical values and the corresponding scientific notations.
Step 3.Write down all unknown and required values,introduce your own notations.
Step 4.THE MAIN PART OF THE PROCEDURE: use your knowledge (refer to Guidebooks if necessary) to create relationships between the known and unknown values resulting into an equation or system of equations (linear,quadratic,trancsendental,differential).
That step forms the MATHEMATICAL MODEL of the given problem.
Step 5.Solve the pure mathematical problem obtained in Step 4.
Step 6.Interpret the solution.Try to explain completely the obtained results (some of them could be senseless).

As a rule the process of working out a apecific AP is preceded by an ALGORITHMIC FLOW CHART MODEL (AFCM),which indicates a computational and logical procedure.The entire process as applied to a certain class of uniform problems exhibits each of the individual parts of the solution together with all the interactions among them.These parts and interactions are indicated by blocks and connecting arrows.Details depend upon the complexity of the class of problems.

Conclusion.

An AP cosists of generalizations,logical sequences,in some cases branching processes.There is a significant distinction between an algorithm and an AP. An algorithm which separates the solution of the problem into very small portions giving exhaustive information inside the steps,and exact sequence of execution so that both,a human and a computer are able to work out all steps in order to obtain the solution,is an absolute predescribed,predetermined, precise instrument,while an AP permits various paths of design, freedom in choosing a decomposition,it presents an application of knowledge,intuition,skills,and scientific background.

A MATHEMATICAL MODEL SOLVING A LINEAR NONHOMOGENEOUS DIFFERENTIAL EQUATION OF THE N-th ORDER WITH CONSTANT COEFFICIENTS