An Introduction to *Mathematica™*

*Stephen Wolfram*

The University of Illinois at Urbana-Champaign

**What Is Mathematica™?**

*Mathematica* is a general system for doing mathematical computation and can be used in many different ways. This section gives a brief survey of some of them.

One way to use *Mathematica* is as a "calculator". You type in a calculation, and *Mathematica* immediately tries to do it. *Mathematica* does far more than a traditional electronic calculator would: as well as numerical operations, *Mathematica* can do symbolic and algebraic operations, and can generate graphics.

When you use *Mathematica* like a calculator, you are drawing on its built-in mathematical capabilities. But *Mathematica* is also a language, in which you can make your own definitions. You can write programs in *Mathematica*, working not only with numbers, but also with symbolic expressions and graphical objects.

You can use *Mathematica* as a language for representing mathematical knowledge. You can take mathematical relations from handbooks and textbooks and enter them almost directly into *Mathematica*. The basic approach is to give a sequence of "transformation rules" which specify how *Mathematica* would treat expressions with different forms.

As well as being a language, versions of *Mathematica* on many computers also serve as complete environments for computation. You can, for example, create "notebooks", which consist of ordinary text, mixed with graphics and "live" *Mathematica* input.

*Mathematica* is set up to fit in with other standard programs. You can use *Mathematica* to prepare input, or analyse output, from external programs. What makes this possible is that *Mathematica* supports many standards, such as UNIX pipes and POSTSCRIPT, which are common to many modern programs.

**Mathematica Is a System for Doing Calculations**

This section gives examples of the three main types of calculations that *Mathematica* can do: numerical, symbolic, and graphical. Each example consists of a short "dialog" with *Mathematica*. The lines labelled IN[n] are what you would type in; the ones labelled Out[n] are what *Mathematica* would type back.

**Numerical Calculations**

At the simplest level, *Mathematica* will do numerical calculations, just like a standard electronic calculator (Figure 1). *Mathematica* can, however, go far beyond a standard calculator. It can, for example, calculate with numbers of arbitrary precision. It can evaluate a wide range of mathematical functions, including all standard special functions of mathematical physics.

*Mathematica* works not only with single numbers, but also with more complicated structures. You can use *Mathematica*, for example, to do operations on matrices. The standard operations of numerical linear algebra are built into *Mathematica*. You can also use *Mathematica* to find Fourier transforms, least-squares fits, and so on. *Mathematica* can do numerical operations on functions, such as numerical integration and numerical minimization.

**Figure 1. Numerical Value of \( \log(4\pi) \)**

**Symbolic Calculations**

The ability to deal with symbolic formulae, as well as with numbers, is one of the most powerful features of *Mathematica* (Figure 2). This is what makes it possible to do algebra and calculus with *Mathematica*. 
Example: Find a formula for the integral $\int \frac{x^4}{(x^2 - 1)} \, dx$.

Here is the expression $x^4/(x^2 - 1)$ in Mathematica.

\[
\text{In[1]} := \text{Integrate}[\frac{x^4}{x^2 - 1}, x]
\]

This tells Mathematica to integrate the previous expression. Mathematica finds an explicit formula for the integral.

\[
\text{Out[2]} = \frac{\log(x + 1)}{2} - \frac{\log(1 - x)}{2} - \frac{\log(x + 1)}{2} + \frac{\log(x - 1)}{2}
\]

Figure 2. A formula for $\int \frac{x^4}{(x^2 - 1)} \, dx$

Mathematica does many kinds of algebraic computations. It can expand, factor and simplify polynomials and rational expressions. It can solve polynomial equations, or systems of such equations. It can get algebraic results for many kinds of matrix operations.

Mathematica can also do calculus. It can evaluate derivatives and integrals symbolically. It can derive power series approximations.

Graphics

Mathematica does both two and three-dimensional graphics (Figure 3). You can plot functions or lists of data. The three-dimensional graphics that Mathematica produces can be quite realistic: they can, for example, include shading, color, and lighting effects.

You can use Mathematica to make two- and three-dimensional pictures. You supply a symbolic representation of the objects, say polygons, in the picture, and Mathematica will produce a graphical rendering of them. Mathematica also has powerful animation capabilities.

Example: Plot the function $\sin(xy)$ for $x$ and $y$ between 0 and $\pi$.

The function \(\sin(xy)\) is a function of $x$ and $y$. There are many options for controlling graphs in Mathematica.

\[
\text{In[1]} := \text{Plot3D}[\sin(x y), \{x, 0, \pi\}, \{y, 0, \pi\}]
\]

Figure 3. The Graph of $\sin(xy)$ for $0 \leq x, y \leq \pi$

Mathematica is a Programming Language

You can write programs in Mathematica, must as you would in a language like C (Figure 4). Mathematica is an interpreter: you can run your programs as soon as you have typed them in.

Example: Plot a table of the first $n$ prime numbers.

This defines function $f$ which makes a table of the first $n$ prime numbers.

\[
\text{In[1]} := \text{Table}[\text{Prime}(i), \{i, n\}]
\]

\[
\text{Out[1]} = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}
\]

Figure 4. A Program in Mathematica

Mathematica Is a System for Representing Mathematical Knowledge

Mathematica gives you a way to represent, and use, the kind of information that appears in tables of mathematical formulae.

Fundamental to much of Mathematica is the notion of "transformation rules", which specify
how expressions of one form would be transformed into expressions of another form. Transformation rules are a very natural way to represent many kinds of mathematical relations.

Some relations that you could use to define your own logarithm function in Mathematica are given in Figure 5.

<table>
<thead>
<tr>
<th>Mathematical form</th>
<th>Mathematica form</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log(1) = 0 )</td>
<td>( \text{Log}[1] = 0 )</td>
</tr>
<tr>
<td>( \log(e) = 1 )</td>
<td>( \text{Log}[e] = 1 )</td>
</tr>
<tr>
<td>( \log(z) = \log(x) + \log(y) )</td>
<td>( \text{Log}[x, y] := \text{Log}[x] + \text{Log}[y] )</td>
</tr>
<tr>
<td>( \log(z^n) = n \log(x) )</td>
<td>( \text{Log}[x^n] := n \cdot \text{Log}[x] )</td>
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</tbody>
</table>

Figure 5. Relations to Define the Logarithm Function in Mathematica

Mathematica Is a Computing Environment

Mathematica gives you an environment in which to set up, run and document your calculations and programs.

There are two pieces to Mathematica on most computers: the “kernel”, which does computations, and the “front end”, which deals with user interaction. (These pieces do not necessarily have to be running on the same computer - you can use Mathematica over a network.)

The kernel works in the same way on all computers that run Mathematica. The front end, however, is set up to take advantage of the different capabilities of different kinds of computers.

On the Macintosh, for example, the Mathematica front end lets you use graphical tools to manipulate your input and output, and to insert extra text.

The Mathematica front end on the Macintosh takes advantage of the Macintosh’s graphical capabilities. Many Mathematica front ends support “notebooks”, which contain a mixture of text, graphics and Mathematica definitions (Figure 6).

Figure 6. A Mathematica Front End Example

There is a growing library of Mathematica notebooks which serve as “live textbooks” on a variety of different topics. You can read the text in a notebook to learn about a topic, and then use the Mathematica definitions in the notebook to do calculations.

A part of a Mathematica notebook about orthogonal polynomials is given in Figure 7. The material in the notebook is arranged in a hierarchical fashion, so you do not need to see details unless you want to.

Figure 7. Part of a Mathematica Notebook about Orthogonal Polynomials
Mathematica is a Tool in the Standard Computing Environment

Mathematica interfaces to many elements of standard computing environments. Under UNIX, Mathematica can communicate with external programs through pipes. You can use output from Mathematica as input to many kinds of programs. Mathematica can write out expressions to use as input for C or FORTRAN programs. It can also produce input for TeX, which you can use to typeset papers and presentations.

Mathematica produces graphics using POSTSCRIPT. You can render the POSTSCRIPT on many different kinds of devices. You can also read it into other programs.

The Mathematica Book


- Hardcover version (ISBN 0-201-19334-5): $44.25

To order the book directly, call Addison-Wesley at 1-800-447-2226.

Buying Mathematica

Mathematica for the Macintosh. There are two versions of Mathematica for the Macintosh:

- Standard Version for Macintosh Plus, SE and II: $495
- Macintosh II Version for Macintosh II only: $795

The only difference between the two versions is speed. The Standard version will run on a Macintosh II, and supports color, but does not make use of the Macintosh II 68881 numeric coprocessor, and is consequently several times slower than the Macintosh II version for operations such as graphic rendering.

Note: Both versions of Mathematica for the Macintosh require 2.5 megabytes of RAM. We recommend 4 megabytes or more.

To buy Mathematica for the Macintosh, visit your local software dealer, or call Wolfram Research at 217-398-0700 (for orders only: 1-800-441-MATH).

Mathematica Front Ends. Mathematica consists of two pieces: a kernel that does computations, and a front end, which handles interaction with the user. You can run these two pieces on separate computers. Thus, for example, you can have the front end on a Macintosh, with the kernel running on a remote computer server.

The Macintosh front end for Mathematica will be made available as a separate product from Wolfram Research. It will run on Macintoshes with only one megabyte of memory. Call Wolfram Research at 217-398-0700 for information.

University Arrangements. Special arrangements are available for universities which distribute a copy of Mathematica with every Macintosh II (or every Macintosh) that they resell. University administrators interested in this program should contact Wolfram Research at 217-398-0700 for further information.

Mathematica for 386-Based MS-DOS Systems. There are three versions of Mathematica for 386-based MS-DOS systems:

- 386 Version (no numeric coprocessor required): $695
- 386/7 Version (287 or 387 numeric coprocessor required): $995
- 386/Weitek Version (Weitek numeric coprocessor required): $1295

MS-DOS 386 Mathematica requires 640k of RAM, and at least 1 megabyte of extended memory. It also requires 5 megabytes of hard disk space.

MS-DOS 386 Mathematica supports CGA, EGA, VGA, Hercules, 8514 and other graphics standards. It supports PostScript, LaserJet, Epson FX and Toshiba P3 compatible printers.

MS-DOS 386 Mathematica includes DOS-style editing capabilities, but does not support Mathematica Notebooks.

To buy MS-DOS 386 Mathematica, visit your local software dealer, or call Wolfram Research at 217-398-0700 (for orders only: 1-800-441-MATH).

Other Systems. Versions of Mathematica are available for the following systems:

- Ardent - for Tital systems
- IBM - for RT/AIX systems
- MIPS - for M/120 systems
- NeXT - bundled as part of standard software on all NeXT Computers
- Silicon Graphics - for Iris 4D systems
- Sony - for NEWS systems
- Stellar - for GS-1000 systems
- Sun - for Sun-3, Sun-4 and Sun-386i systems

These versions are available directly from the hardware manufacturers. Contact your local sales representative for information.

For information on possible other versions of Mathematica, contact Wolfram Research at 217-398-0700.