Calculus, Technology, NSF and the Future

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A Perspective

In the rush of technology developments, it is easy to get the impression that changes are arriving faster than you can think. It is not only easy—it is correct. Before most people had even realized that we had graphing calculators, we had symbol manipulating ones. And before we even started to conceptualize the use of symbol manipulating units in our curriculum, there is over the horizon spreadsheet calculators and who knows what else.

To a generation of mathematicians that remember square root keys as “major break throughs” (Freiden, 1963) and the total surprise of a “hand held scientific unit” (HP35, 1972); the pace and magnitude of the current developments is overwhelming. Yes, things are happening almost at will and the historical chart of dates supports this observation. In Figure 1, we see that it was 2600 years from the abacus to Pascal’s mechanical adding machine. Then 260 years from there to the Monroe mechanical calculator. Another 26 years to electrical devices and perhaps 2.6 years to scientific devices. The sequence is not exact, but the pattern of order of magnitude reductions in the number of years is replicated in the approximate 0.26 years between graphing calculators and symbol manipulating units.

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<th>Calculator Timeline</th>
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Figure 1. Important Calculator Dates

The typical mathematics faculty member notes these passing events and even occasionally participates in philosophical discussions on the implications of technology developments. However, in their day-to-day practices, their courses—yes, even the ones in statistics—avoid any real integration of hand held calculating devices in the course content. We wonder why NCTM and all the professional organizations that focus on the precollege level of mathematics instruction are having such a rough time implementing their calculator recommendations when college level mathematics faculty are turning out to be some of the most conservative practitioners of all.

That will change and there are two observations that support the point. First of all, calculator proficiency is becoming a standard beginning level requirement in engineering schools. In the talks that I have given to mathematics groups, this observation is usually a surprise and quite a shock when they see the level of the expectations. If mathematics instruction is going to hold its own, even in its traditional role of preparing students for science and engineering programs, then major integration of calculators will have to become the norm. To illustrate this, Figure 2 reproduces the calculator proficiency examination that freshmen engineering students at Clemson University must pass out of three trials with scores of 80% or better in order to remain in the College of Engineering beyond their freshman year.

Figure 2. Freshman Engineer Calculator Test at Clemson University

To further support this point, I have very loosely charted the quality level of computer services that faculty and students have received at a typical university over the last two decades in
Figure 3. Usually, faculty members have immediate access to a new service when it is introduced and advanced students usually wait about five years before they receive the same advantage on a regular basis. Then it takes another five years for the undergraduates to routinely receive the service. This pattern was consistent throughout the 60’s and 70’s and it is now collapsing into a uniform level of service to the total university community.

Two significant breaks are worth noting. The initial bend in the undergraduate curve took place when students started receiving improved computer services on a regular basis. This was a result of faculty members realizing that computing was an important part of some undergraduate courses. The second bend is a result of the introduction of micro computers into the secondary schools. Prior to that point, undergraduates arrived on campus and received their computer instruction through variations of several types of batch processes. After all computing was essential to research projects and just tangentially important to undergraduate instruction. All those undergraduates would degrade the interactive systems at most universities and, with the exception of a very few schools like Dartmouth, undergraduates were not given uniform access to the university’s interactive systems. When the new wave of high school graduates arrived on campus with screen editing experiences on their Tandy and Apple micro computers and then “graduated” to the big time of batch systems—all hell broke loose. The result was that the undergraduates would not regress to a lower level of service than they were used to in their high school environment.

That same explosion will take place when the new wave of calculator proficient students arrive on campus and they sit in the college classrooms being served mathematics at a lower level than they received in high school. Imagine the reaction of a “Casio” student watching a mathematics instructor spend thirty minutes finding the graph of a function when he has been sitting with it on his calculator screen from the very beginning. Yes, there are important contributions that college mathematics makes to the understanding of a graph, and we had better make them the focus of our curriculum. We need to accept graphing calculator technology and present college material as devices for understanding the graph instead of items that allow us to draw the graph. Otherwise we will have a growing credibility gap with the students and a deterioration of our programs.

![Typical Campus Computing Facilities](image)

**Figure 3.** Computer service levels at a typical university

**Why Calculus?**

The National Science Foundation recognized the need for significant curriculum changes in collegiate mathematics and through its advisory groups established that an initial effort should be made at the calculus level. There was a high level of dissatisfaction with the current calculus course and a great deal of public debate and interest in “calculus reform.” A modest conference at Tulane University in January, 1986, had produced a report Toward a Lean and Lively Calculus and considerable public debate was being generated by the publication. At the national meetings, panel sessions and talks about the problems in calculus were drawing “overflow” audiences. All this interest was focused at a National Colloquium, “Calculus for a New Century” sponsored by the National Research Council in Washington, DC, in October, 1987. Over 600 faculty members attended and a national crusade was launched.

The new NSF calculus curriculum initiative is a major element in the ongoing developments. In the summer of 1988, the initial 25 awards (Figure 4) were announced and automatically the recipient investigators were thrust into the national spotlight. Their programs are directed toward (i) new topic selection and arrangements, (ii) integration of technology, (iii) the integration of research topics and methods into the course and (iv) the establishment of program linkages and cooperative efforts. Their activities are being widely reported in articles in the AMS Notices, the MAA Focus, and the SIAM News. I will not try to detail the individual projects. One NSF grant was
awarded to all three of the mathematics organizations to establish a publication, *UME Trends*, that will have as its exclusive focus topics in undergraduate mathematics education.

**FY88 Calculus Awards**

**Multi-Year:**
Five Colleges Inc., New Mexico State Univ., Rollins College Colorado School of Mines, American Math. Society

**Planning Grants:**

**Conferences:**
University of Miami

**Figure 4. FY 1988 Calculus Curriculum Awards**

The NSF Calculus Curriculum program is now well established and the proposal deadline for the next year is February 1, 1989. There are several changes in the program. As noted in Figure 4, the FY 1988 awards included many planning grants. In FY 1989, the planning grants have been essentially eliminated, and the thrust of the program will be in full scale curriculum projects. The program is seeking both large and small scale efforts. The large scale projects will be expected to have national impact and represent comprehensive approaches. The small scale projects will still be expected to have national impact, but their focus could be on a more restricted set of considerations. You could loosely describe the difference between the two levels as “mover and shaker grants” and “exploratory pockets.” There are two noteworthy quotes from the current program guide and they each capture very important expectations in the calculus program.

"The typical calculus course captures little of the spirit and excitement of current developments in mathematics and does little to encourage and initiate the mathematics major.” (NSF Calculus Program Guide, p. 1)

"Many of the leaders in the current calculus debate are individuals with international reputations. Their continued advice, counsel and participation in the calculus reform movement are essential." (NSF Calculus Program Guide, p. 3)

Even with all the public debate and discussion, you still encounter the basic question, “Why Calculus?” The selection is best explained with observations about the future need for individuals in the technical work force and the changing demographics of the student population. Everyone recognizes the increasing level of technology sophistication and especially the expansion of the mathematical content of essentially every professional field. Generally this recognition is acknowledged and everyone proceeds to presume that the nation will meet its work force needs with an expansion of its usual methods. The Brookings Institute reports that the change in the entering work force between now and the turn of the century will be only 15% “white male” – yes that’s right 15%. When you realize that the typical high school graduate in that group is now beyond the first grade, we see that radical changes must be made immediately. Otherwise, the nation will continue to try to meet the majority of its technical work force needs from the white male population, and the task is impossible. The long time advocates of educational reform that have been motivated by moral and ethical considerations are now joined by the practical considerations of economic survival.

Calculus plays a key role in these considerations. We know that in general terms, from high school to the Ph.D., we lose 50% of the mathematics students each year. However, this loss is not uniform and a massive drop occurs at the beginning year of college and at the calculus course. Calculus is the one course that sits in a choke point position (Figure 5). It is the focus of most of high school mathematics and is the door way into essentially all of the technical careers in college.

The basis of the NSF consideration is that a major curriculum thrust at this important spot in the curriculum sequence might have immediate and consequential results. We’ll see!
Other Important National Science Foundation Programs

Mathematics faculty members should look to a variety of NSF programs to support their curriculum needs (Figure 6). As the community, including many members of this audience, develop the curriculum materials that we need to revitalize the calculus, then we will be left with the challenge of delivering the materials to our students in an effective fashion. In many cases that will involve computers and other technology devices. The proper place for the faculty members to look to at NSF is the Undergraduate Science and Mathematics Education Division (USEME) within the Science and Engineering Education Directorate (SEE). Their support includes the important Instrumentation and Laboratory Improvement Program. This is the ILI program, and it is under subscribed by mathematics departments. I would encourage each of you to familiarize yourself with the program and present a well thought out proposal to the next competition. The deadline is usually in November. The project should be curriculum driven. On that basis, tell the reviewer what you would do in curriculum improvement with equipment if you had the money. If you make a convincing case, then NSF will give you half of the funds that you need. Your institution will be required to provide a matching amount. However the program rules are flexible, and you do not need to have the matching funds "in hand" when you make the proposal. This point should be discussed with an NSF program officer in USEME and in general the matching funds requirement is not a handicap. Inertia on the part of the mathematics faculty is a far larger problem. We need to realize that NSF responds to what is called "proposal pressure." Unless the mathematics community tells NSF that good mathematics instruction needs equipment, then the NSF funds will continue to be spent in the other disciplines (Figure 7).

Major changes in mathematics instruction will necessitate major retraining of large groups of instructors. Even with the new materials and new equipment, there still remains the major problem of faculty training. Since much of the curriculum innovation talent and many of the leaders in the usage of technology in the mathematics curriculum are part of the audience at this conference, then I would encourage each of you to get familiar with the NSF Faculty Enhancement Program. These funds give support to curriculum training workshops and institutes for faculty members, and it is the program that we should look to for funds to cover our faculty training needs. Mathematics has been an active participant in this program and a number of very successful programs are a credit to our participation. But we should not rest on our laurels. There is still a lot of need for faculty training.

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**NSF Programs**

**SEE - USEME**

**Calculus Curriculum Development**
February 1, 1989

**Instrumentation & Lab. Improvement**
November 21, 1988

**Undergraduate Faculty Enhancement**
October 14, 1989
March 3, 1989

**Figure 6. NSF Programs**
The Future

I think that mathematics is on the threshold of very significant changes in its basic service curriculum. The impact of technology will be widespread. The secondary school program will be fundamentally changed by the availability of simple, inexpensive graphing calculators with matrix capabilities. The college level program will change as a result of a different class of entering students, as well as the availability of symbol manipulating routines in computing devices. Especially the hand held devices.

Clemson University hopes to be on the leading edge of that change. Through grants, we have offered to a wide base of secondary teachers in our "feeder schools" a program that equips the teachers with Sharp EL-5200 graphing calculators and a full semester course in the proper instructional use of the machines. We feel that graphing technology will have the significant impact that everyone is discussing. However, little debate has been focused on the changes in the curriculum that will result from the ready availability of matrix operations in the calculators. I feel that capability will be equally significant. Students will be able to routinely solve "just in time inventory" problems and a wide slate of exciting mathematical applications of linear algebra. A serious challenge is now presented to the educational community to equip and train secondary teachers in the new opportunities that technology changes make available.

At the college level, Clemson University has a multiyear FIPSE grant to revise our undergraduate service curriculum in mathematics to include the assumption that each student will have an HP-28S calculator. In the pilot developments each student is loaned a HP-28 and in the final implementation, each entering freshman will be required to purchase the unit to use throughout their undergraduate courses in mathematics. Our classroom and student computer labs will have microcomputers for software demonstration and special assignments. We see the desk top and the hand held computers to be units that each have special advantages. In one you have more power, color and a wealth of software. All of these are especially useful in demonstrations and special assignments. In the other hand held units, you have infinite portability and continual presence. Classes and especially tests will routinely call on the powerful capabilities of the machines. It is an exciting time in mathematics.

Already, the NSF calculus grants are beginning to impact the quality of calculus instruction in the colleges and universities. The presentations are still playing to "standing room only" crowds at professional meetings. The NSF program and the national debate has made it respectable again for mathematics faculty members to express an interest and a concern for the level of calculus instruction in their departments. The funded grants will make major contributions, but their results will be overshadowed by the wealth of improvements that are already taking place throughout the departments at large and small institutions.

The basis calculus topics are still fundamental in the background of every educated individual. The presentations and instructional approaches to these topics will change and improve. The manner of how one uses calculus concepts will change, but the quantitative evaluation of change, motion, averaging, and smoothing will remain basic and essential.