

## Lessons from Interdisciplinary Courses

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I have an unusual perspective on changing pedagogy in mathematics because of my current teaching situation. Frequently, half of my teaching load is an interdisciplinary course. As a "master learner", I help students describe, analyze, and evaluate such diverse works as Plato's Euthyphro and Michelangelo's David. For this course to be effective in the way it was conceptualized, it is imperative that most class meetings are interactive and all students participate. Due to this strategy for the course, I have had to discard the lecture method and, in its place, use group work in a variety of formats. I have gradually changed how I teach mathematics classes. Yes, I still lecture, but not always and with modifications. The lessons that I have learned that apply to teaching mathematics are:

- \* provide a variety of classroom experiences,
- \* writing is an effective tool in a mathematics class,
- \* doing mathematics should be viewed as a process.

Provide a variety of classroom experiences. Not only is this more interesting for the student, it is for the professor as well. One change from the lecture format is to use small and large group discussions.

For a unit on problem solving, I list twelve techniques to use in problem solving. Then the students work on problems in groups of three to five. Problems I use from A Mathematics Sampler by Berlinghoff, Grant and Skrien include:

What is the sum of the angles of a decagon?

A piece of music lasting  $22\frac{1}{2}$  minutes is recorded in a single band on a  $33\frac{1}{3}$  r.p.m. phonograph record. When the record is played, the music starts when the needle is 30 centimeters from the center of the record and stops when the needle is 5 centimeters from the center. If the groove containing the recorded music were in a straight line, how long would the line be?

As each group uses the techniques to solve the problem, I am available for consultation. After several examples, the groups become more effective in problem solving.

In upper level classes, I sometimes have the students work in groups to do a proof, usually an exercise in the book being used. One group is then asked to put the proof on the board. The other groups critique the proof.

In a calculus class, to make sure all students are proficient in graphing, I divide the students into groups and have them work on several problems. Each group's goal is that all the students in the group understand how to graph a function. I then draw a name from each group out of a hat and the selected student must graph a new problem at the board. If the student is successful, everyone in the group gets 10 extra credit points. The students work hard so all students end up with the extra credit.

In precalculus, I allot four to five days of the term to group project days. Thanks to the new style books, there are many projects available. As each group works on the project, I am available to answer questions. Students must hand in a typed report when their work is completed. Students enjoy the freedom of the project days and often work in the lounge. Their collaboration draws out their varied skills and enhances problem solving abilities as students learn from each other.

In statistics, for the unit on producing data, small groups each describe an experiment and a sampling design. Each group's work is critiqued by the rest of the students.

A group technique that is unusual for a mathematics class is fish bowl. To force students to think about the linear programming process, I have a group of four students work on a problem, seated before the class. They discuss what the variables are and how to set up the problem. (One student acts as scribe.) There is an extra chair by those of the four students. Any student who thinks he can be of help can come forward and sit in the empty chair long enough to add his input. As the solution progresses, students become excited, knowing they can each contribute to the solution. During the last time I did this, a student's face lit up and she said, incredulous, "I'm having fun - and in a math class."

One of the students' favorite math classes is a viewing of "Apollo 13." At the end of a problem solving unit, the students see the movie in

class. They then write a paper on the problem solving techniques used in the film.

Writing is an important activity in the mathematics classroom. By writing, I don't mean taking notes. A variety of writing activities encourages active learning and thinking about mathematics. With in-class writing assignments, it is important to give students enough time to finish the assignment. For basic assignments, I might wait three to five minutes. For longer assignments requiring thought, I might give students fifteen to thirty minutes.

Sample writing activities are:

Summarize the main ideas of the section.

Now that you have learned the theory, work the problem -

After deriving the cylindrical shell formula, students work an example.

Based on what you know, prove this theorem - given the definition of limit, prove  $\lim_{x \rightarrow 1} (3x-1)$  as  $x$  approaches 1 is 2.

What conjectures seem reasonable for the topic – given the axioms for the Four Line Geometry, students list conjectures they might try to prove.

I have students in beginning mathematics classes keep journals. First, each student writes a letter to her group members telling them how she feels about taking a math course. The students learn they have similar feelings, and this activity helps the student to commit to her group. Later journal entries force students to think about the process of doing mathematics and the meaning of the terms. After a problem solving unit has been done, I ask the students to discuss the problem solving techniques used for problems in later sections.

Group projects are an excellent way to get students writing mathematics. A typed report is the final product for the project. By working together for this goal, students learn from each other.

One of my most effective writing assignments is a paper on Apollo 13 that I require in a Mathematical Ideas course. Each student must write a paper that discusses the problem solving techniques used in the movie. The results have been excellent, the lesson apparently learned. One paper started:

"When I was in high school I used to wonder how my math class was ever going to be useful. I used to ask myself, "Self, when are you ever going to need to bust out the Pythagorean

Theorem to solve an everyday problem?" Little did I know how important math is.

The movie Apollo 13 helped me realize how important the fundamentals of problem solving are. . . . While watching the movie I noticed many of the problem solving techniques from our text being used to, basically, save peoples' lives. WOW! With Math!"

An example in the middle of a paper is:

" First they would identify and clarify the problem. A good example would be when NASA discovered that the crew would run out of oxygen and be overcome by carbon dioxide before they got back to earth. They determined exactly how much oxygen it would take for the astronauts to get back alive . . ."

Another paper ends:

" It is easy to see that several basic problem solving techniques were used over and over again to come up with the solutions that were needed to save the Apollo crew. It is also interesting to see how these same techniques can be applied to math problems as well to make finding a solution to them a much simpler task. But what I found to be most interesting of all was that the most prominently emphasized problem solving technique used during the movie was the technique of team work."

Just as the writing of a paper is viewed as a process that requires editing and reanalysis, writing the solution to a math problem or writing a proof should be viewed as a process that requires editing and reanalysis.

For problem solving, I have used various editing techniques to help students become good problem solvers. I have students work a problem first. Then I do the problem at the board and students can see where they have made errors. Another method is to have the students suggest steps as I work through the problem. Problems where there are traditionally mistakes, such as finding the derivative using the definition, can be collected and edited. Students working in groups become each others editors.

Editing and reanalysis is important for students learning to write proofs. For take-home proof problems, I allow students to talk over editing with me. An editing example for the proof that if  $f$  and  $g$  are injective then  $gf$  is follows:

Try 1. Suppose  $g(f(a))=g(f(a'))$ .

As  $f$  is injective, then  $f(a) = f(a')$  implies  $a = a'$ .

Comment. You don't know  $f(a) = f(a')$ .

Work with  $g$  first.

Try 2. If  $g(f(a)) = g(f(a'))$  then, as  $g$  is injective,  $f(a) = f(a')$ .

As  $f$  is injective,  $f(a) = f(a')$  implies  $a = a'$ .

Comment. Conclusion?

I started using the editing process last year and proofs on in-class exams have been much improved, not only correct but more coherent.

Why should math professors make changes as I have described? The students will be more successful in the courses and learn more. They will also have better attitudes towards mathematics. At the end of one term, I asked what had initially been a very apprehensive group of students to write a journal entry titled "Has it been all that bad?" Some responses are:

"No. In fact, I don't hate math anymore. I wouldn't go so far as to say I like it, but I don't hate it. For the first time, I feel I am capable of analyzing a problem and making a logical attempt to solve it. I don't feel like I have to give up if I don't get it the first time."

"It has been quite the contrary! I am so proud of myself for what I've learned in this class, and at the same time, I'm feeling a lot of regret. It's a shame I have dreaded taking any college level math courses for over 25 years

...

So here I am at 45 taking a math course and really enjoying it. I can hardly believe that I am getting A's. My grades reflect the hard work I've put into studying, but the pride I feel for finally feeling comfortable with numbers is beyond description... "

"Simply put, no! I found it interesting, yet comforting that many people struggle through math like me. I cannot ever remember getting an A on a math test until this class. I certainly do not imply that it was easy. It wasn't. . . . Furthermore, I have learned some techniques, such as problem solving and probability rules, that I will be able to use in my job. In closing, I thought the teacher really helped me learn."

Note the last response. The emphasis in the classroom should be on learning not teaching.