

Extending Calculus Reform to Prealgebra¹
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by

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Abstract: The paper discusses construction of modeling type problems on topics in Prealgebra - percent, signed numbers, equations, ratio and proportion that connect systematically elementary and advanced concepts extending "Calculus Reform" to Prealgebra. The approach has been class-tested for three semesters starting Spring 1996 by more than 400 students each semester.

Introduction: During the intersession of January 1996, I took upon myself to develop lab problems for Prealgebra - the first remedial course - offered by our department every semester with an enrollment of about 400 students. I am a member of the Basic Skills Committee of the Mathematics Department of my college, Medgar Evers College, Brooklyn of the City University of New York. We wanted problems satisfying the following conditions.

The problems that:

1. Can be done using a scientific calculator/computer. By teaching students to use scientific calculators has advantages in addition to learning technology. They can also have a better understanding of the order of operations.
2. Can be effectively done in a collaborative setting. I have been using the collaborative approach for the last seven years for all courses from Prealgebra to Multivariable Calculus which I teach.
3. Implement NCTM and AMATYC standards. AMATYC standards for content and pedagogy are given in Appendix 1.
4. Are fundamental, challenging, and address students' difficulties systematically. Most currently available textbooks do not teach arithmetic in detail and do not show how algebra is actually generalized arithmetic.
5. Extend the calculus reform movement of introducing a concept where possible using graphs, numbers and formulas. My fascination of the Calculus reform and teaching the Calculus and Multivariable Calculus courses with the CCH texts inspired me to take up this project.

Importance of Item No. 5, i.e. extending calculus reform to Prealgebra, cannot be emphasized enough. The students and society, in general, are accustomed to receiving and analyzing any information in terms of numbers and graphs as can be seen even from the daily newspapers. By bringing in algebraic notation one can synthesize, generalize and form a mathematical model that has far reaching consequences. Moreover, the mathematical concepts at hand can be made relevant, concrete and practical by using graphs and numbers. So the ideas of calculus reform cannot remain limited to calculus alone.

Logistics of the Design of Problems: In our committee we first laid down the sequence of the topics to be taught that implement, in general, the AMATYC standards. The list of topics is given in Appendix 2. Basically, we wanted to introduce Whole Numbers, and Graphs, Decimals and

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two types of Percent Problems, which are applications of decimals, in the beginning of the course. These topics were to be followed by Order of Operations, Geometrical formulas on perimeters and areas as applications of substitution, Metric system, Commutative, Associative and Distributive laws, Combining like terms, Integers, Equations and Problems on percent as applications of equations, Numerical and Algebraic fractions, Fractional Equations. Building formulas from a numerical pattern was also an important goal.

The 4-hour Prealgebra course met usually three times a week, one of which was to be the lab hour. We wanted five problems per lab hour. I felt that for the students to be able to solve the problems in the lab, they should have practice of that type of problems in their two class periods with the instructor also. So ten problems per week for classwork had to be created. In this manner it was necessary to have available a minimum of 195 problems for a 13-week semester. In January 1996 and the Spring 1996 semester I developed more than 200 problems for the Prealgebra course. As I started developing the problems in the spirit of reformed calculus or item No. 5, I realized that they easily implemented the other four conditions. All Prealgebra - MTH 002 - instructors and students in the college have used these problems for the last three semesters. They have responded to the problem sets enthusiastically. We have included these problems in the upcoming revision of the textbook *Prealgebra*, which Dr. Berenbom and I developed, to be custom published for our College. The first edition of the book was published by Hartcourt Brace in 1991.

Important Consequences:

1. Introducing percents and graphs early in the course gives an amazing range of modeling type problems.
2. Change is the essential part of life. In a calculus reform studying that change has been the main emphasis. (Of course Calculus is itself the study of the rate of change.) A similar situation is true in the extension of calculus reform to Prealgebra. For example, instead of finding the percent of a static problem, we study the effect of a series of changes in interest rates. This becomes more meaningful to students. Naturally, you will see lots of tables and graphs in the problem sets.
3. Reflecting on the problem sets I had developed I noticed that through the modeling type problems, concepts so called remedial and isolated have been shown a deeper connection to higher concepts and how they form the warp and weft of the entire fabric of mathematics. For example, take the concept of average. In a normal prealgebra course, the concept of average is presented by taking a collection of numbers, adding the numbers and dividing by the total number of items in the collection. But these problem sets show how the concept leads to the important concepts of slope and rates.

Bibliography that helped me create the problem sets is enclosed.

In the following I would like to present a selection of the problems based on different concepts involved in the course.

Note: In all problems indicate the answer using appropriate units and not just a number.

Week 1

Concepts and skills: Place Value System, Operations on Whole Numbers, Bar and Line Graphs

1. A chef, who weighed 197 pounds, went on a diet. The first week the chef lost six pounds. During the five subsequent weeks, he lost two pounds each week.

(a) Complete the following table.

Week	1	2	3	4	5	6
Weight	197					

(b) Draw a line graph of weight (in pounds) against weeks. This means marking weeks on the horizontal (x-axis) and weight on the vertical (y-axis) and plotting a point for corresponding week and weight.

(c) How many pounds did he lose altogether?

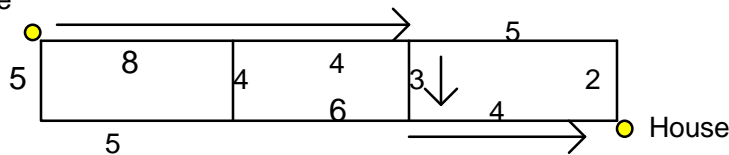
(d) What was his weight at the end of five weeks?

(e) By extending the graph answer the following:

If he continued dieting and lost weight at the same rate as the previous week, what will be his weight at the end of seventh week?

2. A pizza delivery van has to cover the distance from the store to a customer's house with a choice of several streets. Time, in minutes, needed to cover each street is shown.

Store



(a) If the delivery van takes the route indicated how much time is elapsed?

(b) How many other possibilities does it have to cover the same distance in the same amount of time?

(c) Which path from the store to the customer's home would take the longest time?

3. A retailer wants to make an \$87 profit on a TV that he paid \$214 for.

(a) What should be the selling price?

(b) Make a table for # TV's and profit in dollars by filling in the blanks using addition.

# TV's	1	2	3	4	5	6	8	10	12	20
Profit (\$)	87									

(c) What is his profit if he sells 22 TV's of that model? Find the answer using only the table in

(b). You can get the answer in many ways.

(d) Draw a line graph using the table in (b).

4. Mary is a salesperson at Great Discounts Auto Store. The following table summarizes her sales for the last four weeks.

Week	Sales in \$
1	\$4,050
2	1,075
3	900
4	2,425

- (a) How much did she sell for the four weeks?
- (b) Fill in the following blanks and circle "increase or decrease."

Week	Sales in \$	Weekly Change
1	\$1,050	xxxxxxxx
2	1,075	\$-----, increase/decrease
3	1900	-----, increase/decrease
4	2,425	-----, increase/decrease

- (c) Draw a bar graph and a line graph of Sales against Weeks.
- (d) What was the change in sales from the first week to the fourth week.

5. A Beverage and Snacks Company has income from sales from 1991-1996 as in the following table. (Additional goal of the problem: Using appropriate scales)

Year	1991	1992	1993	1994	1995	1996
\$inMillions	17500	19300	21900	25000	28700	29400

- (a) Draw a vertical bar graph using the above table.
- (b) Draw a line graph using the above table.

Year	1991	1992	1993	1994	1995	1996
\$ in Millions	17500	19300	21900	25000	28700	29400
Yearly Increase	xxxxxx					

- (c) Fill in the blanks named yearly increase from 1992 to 1996 based on the previous year's performance. During which year did the company's sales increase the most?
- (d) What was the average yearly increase from 1991 - 1996? What was the average for three years from 1994 to 1996?

6. A \$20,000 investment will grow, depending on your tax bracket as in the following table. Tax Deferred means tax will be paid after 5 years.

Years	0	1	2	3	4	5
20% tax bracket	20,000	21,100	22,300	23,500	24,800	26,200
Tax Deferred	20,000	21,400	22,900	24,500	26,200	28,100

- (a) Draw line graphs of Yearly Balances against Years for each investment.

(b) Complete the blanks in the following tables.

Years	0	1	2	3	4	5
20% tax bracket	20,000	21,100	22,300	23,500	24,800	26,200
Yearly Increase	xxxxxx	_____	_____	_____	_____	_____

Years	0	1	2	3	4	5
Tax Deferred	20,000	21,400	22,900	24,500	26,200	28,100
Yearly Increase	xxxxxx	_____	_____	_____	_____	_____

7. Complete the following square using numbers 2 through 10, only once, so that total horizontally, vertically and diagonally is 18. Some boxes have already been filled to get you started.

		9
	10	

Week 2

Topic/s and Skills: Decimals,. Operations on Decimals. Rounding and Estimation

1. Pam drove her car at 55 mph.

(a) In 4 hours, how much distance does she cover?

(b) Fill in the blanks in the following table. (Note that Distance = Speed x Time)

Time (hours)	0	1	2	4	6	8
Distance (miles)						

(c) Draw a line graph using the table in (b).

2. The Brown family drives a car for a distance of 300 miles from Albany to Buffalo at a speed of 60 mph with a one-half hour lunch break in Syracuse which is 150 miles away from Albany.

(a) Complete the given table.

Time (hours)	0	1	2	3	3.5			
Distance (miles)								

(b) Draw a line graph of distance against time.

(c) What was the average speed for the entire trip?

3. A cab driver drives 800 miles per 6-day week.

(a) What is his daily average?

- (b) If he drove 300 miles for first two days, what is his average daily mileage for the first two days? What was the average for the last four days?
 (c) Make a table of distance covered per day using the information from (b).

Day	0	1	2	3	4	5	6
Distance(miles)							

- (d) Draw a line graph of distance against days from (c).

4. An investment of \$100 doubles every 4 years.
 (a) How much will be the balance at the end of 16 years?
 (b) Fill in the following table.

Years	0	4	8	12	16
Balance (\$)					

- (c) Draw a line graph of balance against years.

5. A parking garage charges \$6 for the first hour. The hourly rate for parking after the first hour is \$3, with the maximum rate of \$20. Partial hours are rounded to the next hour.
 (a) If a car is parked for 2 1/2 hours, what will be the parking charges?
 (b) If a car is parked for 3 hours 40 minutes, what are the parking charges?
 (c) Complete the following table.

Hours	1/2	1	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	6
Parking charges											

- (d) Draw a graph of parking charges for the first hour. Draw a graph for the second hour. Repeat the process for 6 hours. Describe in your words how the graph appears.

6. A video store has a starting expense of \$5,000.
 (a) If the store rents \$3 per tape, how many tapes should be rented to break even?
 (b) If a profit of \$3,000 is made, how many tapes were rented?

7. Every week Joe earns \$15 per hour for the first forty hours, and \$23 per hour overtime.
 (a) If he earned \$830 one week, how many extra hours did he work that week?
 (b) Complete the following table.

Hours	10	20	30	40	45	50	60
Wages (\$)							

- (c) Draw a line graph from 0 to 40 hours. Draw a line graph for work for more than forty hours using the above table, starting at 40 hours and \$600.

8. A plant 10 inches long grows 1.5 inches every week until it reaches 16 inches.

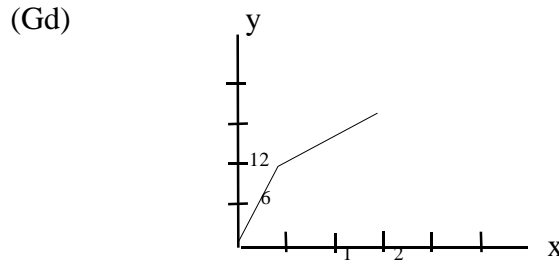
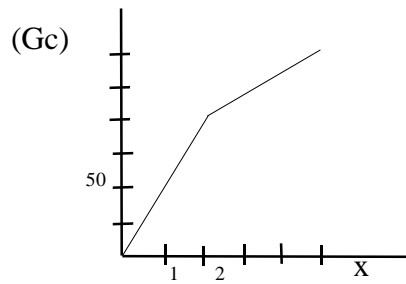
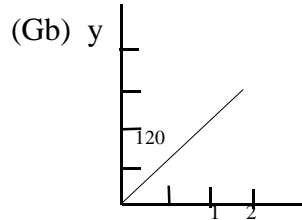
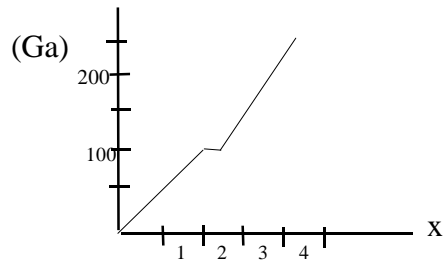
- (a) Complete the following table and using the table, draw a line graph of the height of the plant against time in weeks.

Week	0 (start)	1	2	4	6	8	12
Height (inches)	10						

- (b) What is the height of the plant at the end of 7 weeks? (Answer the question using the graph.)

9. Match with each description in (a) through (d), the appropriate graph in Ga - Gd and then fill in the blanks in (a) through (d).

- (a) Sammy drove at the speed of 60 mph for 180 miles. He drove for ____ hours.
 (b) Suzanna drove at the speed of 50 mph for two hours, took a 30-minute break in the rest area and then drove at 65 mph for two hours. She drove a total distance of ____ miles.
 (c) Carlton rode his bike and then walked for half hour. He rode his bike at the speed of ____mph, he walked at the speed of ____mph and he covered a total distance of ____miles.
 (d) Betty went by car at 50 mph for two hours and then took a bus for the rest of the way. She went ____miles by car. She rode the bus for ____hours. The speed of the bus was ____mph.



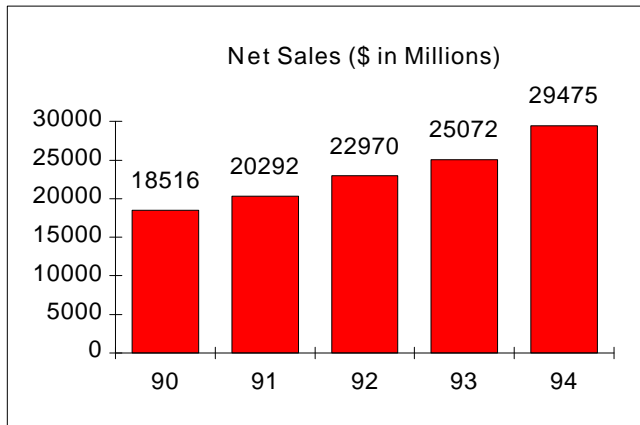
Week 3

Concepts and Skills: *Changing fractions to decimals, decimals to fractions. Changing percent to decimals and fractions, Changing fractions or decimals to percent. Finding percent of a number*

1. Complete the following table based on data from the census of a village.

Ages	0-17	18-29	Over 30	Total
# males	320	500	1800	
# females	210	750	2000	
Total				
Fraction representing male population				
% of male population				

2. The following chart is from an annual report of a corporation.



Using the above chart answer the following questions.

- (a) What was the increase in sales from 1990-1991. Find the percent increase. (Hint: To find % increase for 1991. find the fraction: Increase from 1990-1991/Sales in 1990. Express your answer as a percent.)
- (b) Complete the following table.

Year	1990	1991	1992	1993	1994
Sales					
Yearly Increase	xxxxxxx				
% Increase	xxxxxxx				

- 3. During a semester Malcolm scores 38/40, 45/50, 32/40, and 36/40 in his 4 exams. What was his percent score in each exam? What was his average for the semester?
- 4. A population survey of 900 people about the job performance of the President was taken. The results are given in the following table, with some information missing. Fill in the missing information, and answer the questions (a) through (c).

	Male	Female	Total
Approve	285		680
Disapprove		120	
Total			900

- (a) What percent of male population approve the job performance?
 - (b) What percent of female population disapprove the job performance?
 - (c) What percent of population approve the job performance?
5. A beverage company's profits in December 31, 1994 and December 31, 1995 are given in the following table. Complete the following table. This table will be part of the annual report for the company for its share holders. (Dollars in millions.)

	December 31, 1995	December 31, 1994	Percent Change (+/-)
Beverages	1,567	1,150	
Snack foods	1,235	1,025	
Restaurants	640	724	
Total			

6. One semester Cathy takes three 4-credit and two 3-credit courses. She gets 2 A's in a 4-credit courses, 1 A in a 3-credit course. She gets a B in the remaining courses. The grade of an A is worth 4 points and a B is worth 3 points.

(a) Complete the following table and based on the table, answer (b) and (c). To fill in the total multiply the three entries in the respective column.

	A	B
# 4-credit courses		
Points per course		
# Credits per course		
Total points in 4-credit courses		

	A	B
# 3-credit courses		
Points per course		
# Credits per course		
Total points in 3-credit courses		

(b) What is the total number of points Cathy gets for the semester?

(c) Find her grade point average (GPA) for the semester. Round to one decimal place.

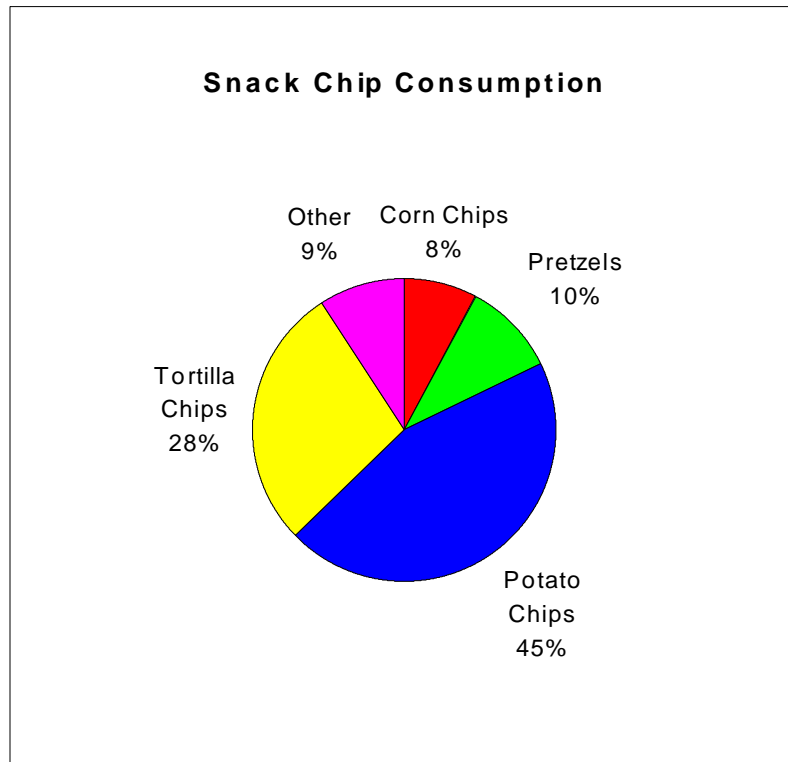
7. Potato and other snack chip consumption (in “Lbs per Capita” meaning pounds per person) in the countries around the world is given in the following table.

Country	US	Canada	United Kingdom	Australia	Mexico	Other
Chip Consumption	18	9	8.5	6	3	.2

(a) What percent of Australia's chip consumption is US chip consumption?

(b) What percent of all the chip consumption around the world is US chip consumption?

8. A snack company's profits are given in the following Pie-chart. If the company's profits were \$125,000,000 last year, how much profit was made from Potato and Tortilla chips?



9. The following chart indicates the next monthly payment on a loan of \$1,000 at different interest rates per year (APR).

(a) Complete the following table.

Annual Rate (APR) (%)	6	9	12	18	21	24
Monthly rate (APR/12)(%)	0.5					
Monthly payment (\$)	5					

10. A salesperson gets a salary of \$500 per week and 3% commission on all the sales she makes. If she sold \$52,000 amount of merchandise in one week, what was her total income that week?

Week 4

Concepts and Skills: *Exponents, Order of Operations, Variable Expressions, Perimeter, Area, Volume*

1. In a business investment doubles every six years.

(a) Complete the following table and draw a smooth line graph of balance against time in years.

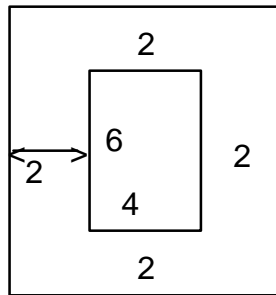
Time (in years)	0	6	12	18	24	30
Balance (\$)	1,000	2,000				

(b) Divide the balance by the initial investment and enter into the blanks titled "Ratio". Write the answer in exponential form and also as a number.

Time (in years)	0	6	12	18	24	30
Balance (\$)	1,000	2,000				
Ratio	1					

(c) Draw a graph of Ratio against time.

2. (a) A photograph is surrounded by a 2"-wide frame. What is the area of the glass to frame the photograph?



(b) Complete the following table.

Size of picture (in inches)	4" W x 6" L	4" W x 6" L	6" W x 8" L	6" W x 8" L	9" W x 11" L	9" W x 11" L
width of frame	1"	2"	1"	2"	1"	2"
Area of glass						

3. The formula for converting kgs into pounds (lb) is given by: $1 \text{ kg} = 2.2 \text{ lb}$
Complete the following table and plot a line graph of lb. against kg.

kg	1	2	6	8
lb.				

4. The formula $F = 1.8 C + 32$ relates to converting Celsius to Fahrenheit where C is temperature in degrees Celsius and F is the temperature in degrees Fahrenheit.

(a) If the boiling point of water is 100°C , what is it in degrees Fahrenheit?

(b) Complete the following table.

Celsius	10	20	60	80	100
Fahrenheit					

(c) Draw a graph of Fahrenheit against Celsius temperature.

5. The growth of bacteria in a culture is given by the formula: $N = A 2^t$, where t is the elapsed time in hours, N is the weight of bacteria, in grams. Complete the following table and draw a graph of weight against time, given $A = 100 \text{ g}$

time	1	2	6	8
Weight				

6. A formula for simple interest is given by $I = Prt$, where I is the interest, P is the principal (initial investment or loan), r is the rate of interest per period t is the time expressed as the number of periods. Complete the following table with the interest rate of 8.2%.

P	1,000	2,000	5,000	8,000
t	1	5	2.4	6.2
$I = Prt$				

7. Simone invests \$10,000 in a growth stock and \$5,000 in an equity index stock. The growth stock went up by 35.75% and equity index stock went up by 25.28%. Complete the following table. How much is Simone's investment now?

	Growth	Equity index
Initial Investment	10,000	5,000
Rate		
Increase (\$)		
Current value (\$)		

8. 2 gallons of 40% acid solution is mixed with 3.5 gallons of 20% acid solution. What is the concentration of the mixture? Use the following table to answer the question.

	40% solution	20% solution	Mixture (_____ %)
Volume			
Amount of acid (gallons)			

9. A 16-quart radiator has been filled with 30% antifreeze.

(a) How much pure antifreeze does the radiator contain?

(b) To prepare for colder temperatures some solution needs to be drained out and replaced by pure antifreeze. Complete the following table to find out the resulting concentration of antifreeze in the radiator solution.

Amount drained (qt.)	Pure Antifreeze in (30%) solution	Pure Antifreeze (100 %)	Pure antifreeze in Solution	% solution in the radiator
1	$16 \times .3 - \underline{.3}$ qt = _____	1 qt		
2	$16 \times .3 - \underline{\quad}$ = _____			
4	$16 \times .3 - \underline{\quad}$ = _____			
6	$16 \times .3 - \underline{\quad}$ = _____			

Week 5

Concepts and Skills: *Circles, Signed numbers, addition, and multiplication of signed numbers. Introduction to graphing using positive and negative numbers.*

1. The circumference of a circle is given by the formula: $C = 2\pi r$ where $\pi \approx 3.14$, r = radius of the circle.

(a) If Jason wants to make circles of radii of 5, 10, 20, 40 inches from wire, how much wire should he get? First complete the table in (b).

(b) Complete the following table.

Radius (in.)	5	10	20	40
Circumference (in)				

(c) Draw a graph of the circumference against the radius using the table in (a).

2. The area of a circle is given by the formula: $A = \pi r^2$ where $\pi \approx 3.14$, r = radius of the circle.

(a) Complete the following table.

Radius (cm.)	5	10	20	30
Area (cm ²)				

(b) Draw a smooth line graph of the area against the radius using the above table.

3. The distance of a falling object from the ground is given by the formula:

$$d = -.5gt^2 + 512, \text{ where } g = 32, t \text{ is measured in seconds.}$$

(a) Complete the following table

t (sec)	0	1	2	2.5	5	6
d (feet)						

(b) Interpret the result at $t = 6$ seconds.

(c) Draw a smooth graph of distance against time using the table in (a).

4. The formula $F = 1.8 C + 32$ relates to converting Celsius to Fahrenheit where C is temperature in degrees Celsius and F is the temperature in degrees Fahrenheit.

(a) If the freezing point of water is $0^{\circ} C$, what is it in degrees Fahrenheit?

(b) Complete the following table.

Celsius	-10	-20	-60	-40	0	10	20	100
Fahrenheit								

(c) Draw a graph of Fahrenheit against Celsius temperature.

5. In a country there is a flat tax of 17% over \$30,000. If a family makes less than \$30,000 it gets a subsidy of 17% of the difference between the income and \$30,000. The formula is given by $A = 17\%(I - 30000)$, where A is the amount of tax or subsidy, I is the income. Complete the following table.

Income (\$)	40,000	30,000	20,000	10,000	5,000
Amount (\$)					

6. (a) Complete the following table which gives the absolute values (distances from zero) of numbers.

Number	-20	-10	-5	-4	0	4	5	10
Absolute Value								

(b) Draw a graph of the absolute value against number from the table.

Week 6

Concepts and Skills: *Subtraction of signed numbers, The Commutative and Associative Laws, The Distributive Law, Combining like terms*

1. To test the effectiveness of tutoring on a group of students, a test was administered before tutoring and after tutoring. The following table represents difference of students' scores from the passing mark. Complete the third row of the table showing by how much each student progressed.

Before tutoring	-15	-20	-20	5	0	-5	-10	-10
After tutoring	5	-5	10	25	-5	21	-20	20
Change								

2. If an item is not sold during the first week, a store reduces it by 10% per week. After the item reduces to 60% of the original marked price it is removed from the store.

(a) Complete the following table.

Week	1	2	3	4	5	6
Price (\$)	250					

(b) Write a formula for the price, using the unknown P for price in terms of weeks elapsed.

(c) Draw a graph of Price against Weeks.

3. A car is traveling at 65 mph **from** Washington, DC **to** Pittsburgh, PA, a distance of 300 miles.

(a) Complete the table indicating the **distance from Pittsburgh, PA**.

Time (hour)				
Distance (miles)				

- (b) Write a formula for distance (D) from Pittsburgh in terms of time (T) elapsed.
- (c) Draw a graph of distance against time.

4. A submarine is coming to the surface at a rate of 5 m per minute. Complete the table in (a)
 (a) Find the distance from the surface of water.

t (min)	0	1	2	3	5	6
d (meters)	-50					

- (b) Write a formula of d in terms of t.

5. Jerome drives a motorcycle at 50 mph south on I-95 from Wilmington, DE. Rimi drives a car at 60 mph on the same route after 1/2 hour.

- (a) Complete the following table and find an approximate time that Rimi overtakes Jerome.

Time (hours)	0	.5	1	2	3	4	5
Jerome's distance (miles)							
Rimi's distance (miles)							

- (b) Find formulas for Rimi's distance and Jerome's distance in terms of time, t.
- (c) On the same set of axes draw the graph of Rimi's distance and Jerome's distance against time.
- (d) Find out approximately when Jerome's distance equals Rimi's distance. (i. e. the point of intersection of the two graphs.)

6. Find the sum of each row, column and diagonal in the following table.

$a + x$	$a - (x + y)$	$a + y$
$a - (x - y)$	a	$a + (x - y)$
$a - y$	$a + (x + y)$	$a - x$

7. A round-trip from Newark to Miami Beach is \$175 and cost of a hotel room per day is \$55.

- (a) Complete the following table

# Days	1	2	3	4	5	6
Total cost (\$)						

- (b) Find the total cost for a week. Find a formula for the total cost for a stay of x days.
- (c) Graph the total cost against number of days.

8. Frank invests x dollars in a social choice stock and y dollars in a global stock. During the year, the social choice stock went up by 30% and global stock went up by 25%.

- (a) Complete the following table.

	Social choice	global
Initial Investment	x	y
Rate		
Increase (\$)		
Current value (\$)		

- (b) What is the current value of all his investments
 (c) If he invested \$8,500 in social choice stock and \$1,100 in global stock, use the formula from (b) to compute how much money his investment is now worth.

Week 7

Concepts and Skills: Solutions of equations, Solving equations by addition, division and both techniques, Word Problems

1. The formula $F = 1.8 C + 32$ relates to converting Celsius to Fahrenheit where C is temperature in degrees Celsius and F is the temperature in degrees Fahrenheit.

(a) Complete the given table.

City	New York	Chicago	San Francisco	Los Angeles
Temperature ($^{\circ}$ F)	23	15	57	72
Temperature ($^{\circ}$ C)				

(b) Solve the above formula for C.

2. A \$20,000 investment grew for the last four years as given in the table. Find the yearly increase and complete the table if the investment grows at the same annual percentage rate (APR).

Years	0	1	2	3	4	5
Year-end Balance	20,000	24,000	28,800	34,560	41472	
Yearly Increase	xxxxxx	_____	_____	_____	_____	_____
Yearly % increase						

3. If the cab fare is \$25, and Mary has 2 ten-dollar, 4 five-dollar and 8 one-dollar bills. Complete the following table that indicates the bills she can combine to make the fare \$25.00.

ten-dollar bill	five-dollar bill	one-dollar bill	Total Value
2	0		25
1		7	25
0	x		25
	0	x	25
x		5	25

4. Pam starts driving her car at 65 mph from Chicago to Cincinnati, a distance of 295 miles.

(a) Fill in the blanks in the following table. (Note that Distance = Speed x Time)

Time (hours)	0	1	2	t
Distance (miles)	0			

- (b) Draw a line graph of the table in (a).
- (c) Write an equation of distance in terms of time t . Use the equation to find the time needed to cover 200 miles. Verify this fact from the graph in (b).

5. A salesperson makes \$275 and 10% of all sales per week..

- (a) Complete the following table that describes her salary per week..

Sales (\$)	0	1000	2000	x
Weekly Salary (\$)	275			

- (b) One week her salary was \$850. What was the amount of sales she made that week.

- (c) Graph salary against sales. Answer the question (b) using the graph.

6. The relationship between miles and kilometers given by: 5 miles = 8 km.

- a. Find a formula for miles, y , in terms of kilometers, x .
- b. Convert 40 km into miles using the formula in a.

Week 8

Concepts and Skills: Solutions of equations, Solving equations by addition, division and both techniques, Word Problems

1. A population survey of 900 people about the job performance of the President was taken. The results are given in the following table, with some information missing.

- (a) If 42% of the approving population is male, fill in the missing information in the table.

	Male	Female	Total
Approve	240		
Disapprove		120	
Total			900

- (b) What percent of male population surveyed was male?

2. Every week Joe earns \$15 per hour for the first forty hours, and \$23 per hour overtime.

- (a) If one week Joe worked x hours, where $x < 40$, find his wages.
- (b) Complete the following table and draw a line graph. Assume $x > 40$ in this case.

Hours	10	20	40	45	60	x
Wages (\$)						

- (c) If one wee Joe earned \$876, set up an equation for the wages in terms of x .
- (d) During the week Joe earned \$876, how many hours did he work?

3. A 10-inch long plant grows 1.5 inches every week until it reaches 48 inches. Complete the following table and using the table, set up an equation to find the week the plant reaches a length of 48 inches.

Week	0	1	2	4	5	x
Height (Inches)	10					

4. The perimeter of a rectangle is given by the formula: $P = 2L + 2W$. A rectangle has perimeter = 184 cm; W is its width and the length is 8 cm longer than the width.
- Find the length in terms of width, W .
 - Substitute the value in (a) in the given formula for P . Solve the equation for W .
 - Find the width and length of the rectangle.
5. Measurements of a tree are taken at different heights from the ground and is listed in the following table. By assuming the tree to be a stack of cylindrical slabs of height 2 feet, find the volume of the tree as follows.
Find the radius at the given heights.

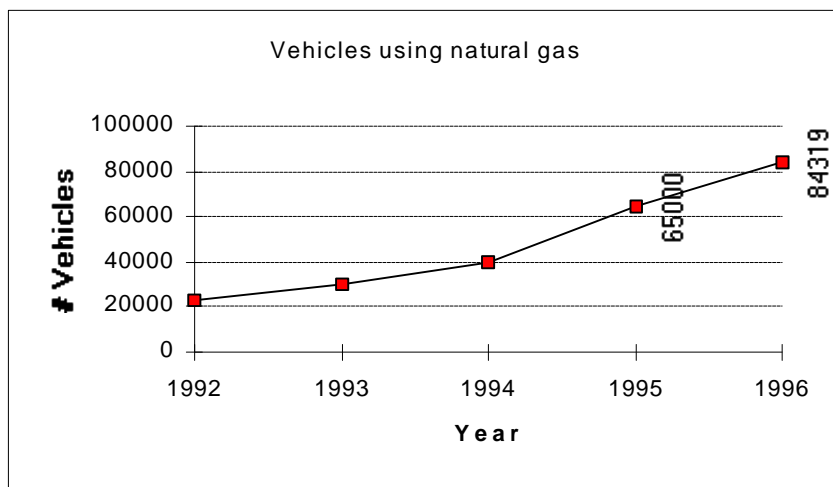
Height from the ground (feet)	0	2	4	6	8	10
Circumference (inches)	35	28	21	14	10	7
Radius (inches)						

(a) Complete the following table.

Radius (inches)						
Radius (feet)						
Area of cross-section (square-feet)						
Volume of slab of ht. 2ft. (cubic feet)						

Total volume = ____ cubic feet.

6. The line graph below lists the number of vehicles using natural gas. What was the (percent) rate of increase for the year 1995-96. If the rate of increase is the same, estimate the number of vehicles in 1997. (Source: The Natural Gas Vehicle Coalition, Energy Department)



7. A credit card company charges 1.5125% monthly finance charge for any unpaid balance. If the finance charge was \$80, how much was unpaid balance?

8. The average price of a home in a city went down to \$85,000 by 8% during the year 1995. What was the average price at the end of 1994?
9. Area of a rectangle is given by the formula: $A = LW$, where L is the length and W is the width. If the area of plywood board is 36 m^2 , and the width is 4 m, find the length of the board.

Week 9

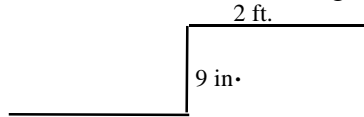
Concepts and Skills: Fractions (Rational Numbers), Rational Expressions, Reducing, Mixed Numbers and Improper fractions, Operations on Fractions and Rational Expressions, Operations on Mixed Numbers

1. A ball dropped on the floor bounced 5 times before it was caught. Every time the bounced height was reduced by half the previous height. The first time it was dropped from the height of 12 feet.
 - (a) List the height the ball reached in each bounce. Answers should be fractions.
 - (b) What was the total distance it covered in five bounces from the time it was dropped.
2. Susan goes to college 15 miles away from home. For $\frac{4}{5}$ of the distance she goes by car at 50 mph, and the rest of the trip she makes by bus at 30 mph. How much time does she take for the entire trip?
3. One number is $\frac{2}{3}$ of another. Call one number X and the other Y . Complete the following table, where the first row is already filled.

X	Y
3	2
6	
9	
12	

- a. Draw a line graph where X is on the horizontal axis and Y is on the vertical.
 - b. Extend the graph and then find the value of X when $Y = 15$.
4. A shampoo manufacturer adds $\frac{1}{3}$ more to the regular 15-ounce bottle without changing the price. What is the new volume of shampoo? If the price was \$1.50 per bottle, what is the change in price per ounce?
5. A filtering pipe 20 feet long reduces pollution by $\frac{1}{3}$ when water is discharged through it. If water with 81 mg of pollutants is filtered through 40 feet of such a filtering pipe, how much of the pollutants are filtered out?
6. Tap water and boiling water are mixed to form a 12-gallon mixture. $\frac{1}{3}$ of the mixture is tap water.
 - (a) What fraction is boiling water?
 - (b) How many gallons of tap water is used?

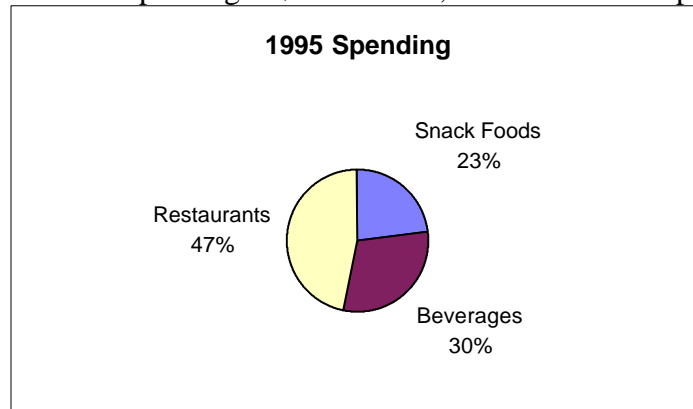
7. There are fifteen steps on a flight of stairs. Each step is two feet wide and 9" high.
- How much height in feet does the flight of stairs cover?
 - What is the horizontal distance does it cover?
 - Find the ratio of width to height.



8. Two baseball teams have the following records:

	Team 1	Team 2
Won	16	20
Lost	20	24

- Which was a better team? Why?
 - What was the percentage of each team? (In sports, percentage is a fraction of number of games won to number of games played, expressed as a three-place decimal.)
9. The following pie-chart describes a company's spending in 1995 building the following three businesses. If Snack Foods spending is \$530 million, what is the total spending?



10. Leisure Video charges \$3.00 to rent a tape. It has a daily expense of \$2,500.

- (a) Complete the following table for the profit at the end of the day.

Tapes	Profit (\$)
0	-2500
1	3 - 2500
1000	3*1000 - 2500 =
x	

- Set up a formula for the daily profit in terms of the number of tapes rented.
 - Use the formula to find how many tapes Leisure Video should rent to make a profit of \$1440.
 - Use the table to graph the daily income against the number of tapes rented.
11. A car depreciates \$2020 per year. If the car cost \$16,950 in 1990. Complete the following table.

Year	1990	1991	1992	
# of years	0	1	2	x
Value of the car	16950			

- (a) Let V be the value of the car at the end of x years. Find a formula for V in terms of years, x .
- (b) Graph the value of the car against the number of years.
- (c) Find in which year the car will be valued at \$3,820? Answer the question using the formula and also the graph.

Week 10

Concepts and skills: Fractional Equations, Cross Multiplication, Ratio and Proportion, Word Problems on Fractions

- 1. Johnson family purchased a computer, printer and furniture for \$3,500. They signed up for an easy payment plan of 4 payments: The first three payments are the sum of a 1 ½% finance charge on the unpaid balance plus 30% of 3,500. The fourth payment is the sum of a 1 ½% finance charge on the unpaid balance plus the remaining unpaid balance.
- (a) Complete the following payment schedule for the Johnson family.

Payment #	Unpaid Balance	Finance Charge	Amount of Balance paid	Installment
1				
2				
3				
4				

- (b) How much more does the Johnson family pay on the easy payment plan than it would have if it paid the entire \$3500 at once?

- 2. Fran invests a part of \$15,000 in a social choice stock and the rest in global stock. During the year, the social choice stock went up by 30% and global stock went up by 25%. Her total investment increases by \$4,100.

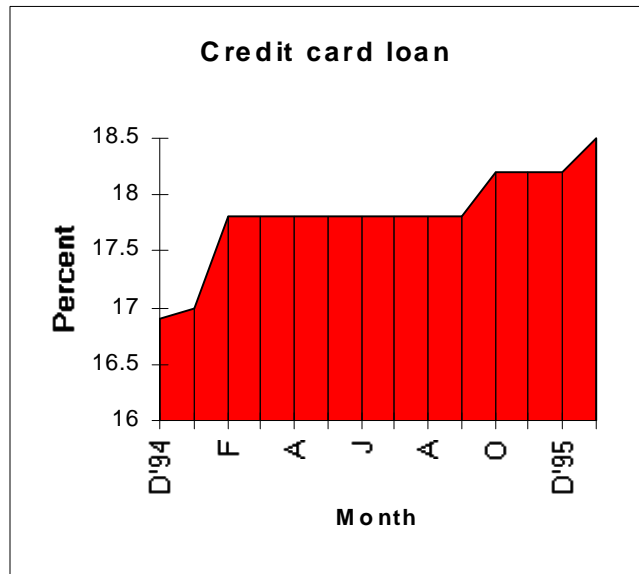
- (a) Complete the following table.
- (b) How much is Fran’s initial investment in each of the stock?

	Social choice	global	Total
Initial Investment (\$)	x		15,000
Rate (%)			
Increase (\$)			4,100

- 3. Current price of a share of W-Mart chain of stores is \$27 ½ and earnings per share is \$.98, while the price and earning per share for L-Mart chain of stores is \$24 ¼ and \$1.45 respectively. Find the ratio of price to earning for each chain of stores.

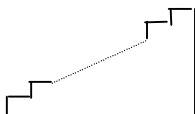
This ratio is called P/E ratio. Which of the above chains has a higher P/E ratio? If you are investing in one of the two chains which chain would you prefer?

4. The following graph describes the credit card APR (Annual Percentage Rate) from Jan. 1994 - Jan. 1996.
 - (a) Find the monthly rate. (Hint: Find APR/12)
 - (b) Make a table to describe the change in credit card rates from month to month by completing the table below the graph.
 - (c) Which month was the change greatest?
 - (d) If \$500 is borrowed on the first day of a month, and returned on the last day of the month how much interest is charged?



Month	12/94	1/95	2/95	3/95	4/95	5/95	6/95	7/95	8/95	9/95	10/95	11/95	12/95
Approx. Rate													
Change over the previous month.													
Finance charge for \$500.													

5. On a photograph of a triangular mosaic, the base of the triangle measures 30 mm and its height measures 24 mm. Actual base of the mosaic is 50 m. How tall is the mosaic? How much area does each of the triangles in the mosaic and slide cover?
6. A rectangular tank is filled in two hours.
 - (a) At this rate what fraction of the pool is filled in twenty minutes?
 - (b) If the tank is 10 ft deep, how much does the height increase per minute?
7. A landing for the 15 feet high upper floor of a 2-story house has to be installed as shown.



One staircase with each step 1 ft wide and 9 inch high has been selected.

- (a) How many steps does the staircase have?
- (b) How far from the wall is the lowest step?

8. 1/4 of the Edwards family income is spent on food, and 1/3 is spent on rent. When the Edwards move to a house their mortgage is 1/2 of the family income which is \$700 more than the rent. What is their total income? How much do they spend for food?

9. In order to increase the number of customers a store-owner has three plans. (1) Run a sale. (2) Advertise more. (3) Give a door prize for first 200 customers. Each plan in combination with another produces profits. The following table describes the cost of each plan and combination she has tried. Complete the last column of the following table.

Run a sale	Advertise More	Door Prizes	Increased Profits	Increased Profits / Expenses
200,000	xxxxxxxxxxxx	10,000	\$1,000,000	
xxxxxxxxxx	500,000	10,000	\$1,000,000	
200,000	500,000	xxxxxxxxxx	\$2,500,000	

- (b) Using the last column in the above table find, which of the combinations should she use more often?

The table as given above is used in the “Design of Experiments”, which is a part of the subject of Statistics.

Week 11

Concepts and skills: Pythagorean Theorem, Denominate Numbers

- 1. (a) Complete the following tables of square roots.

x	1	4	9	16	25	36	49	64	81	100
y = √x		2	3							

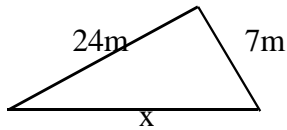
- (b) Use the table (a) to draw a smooth graph where x is on the horizontal axis and y is the vertical axis.
- (c) From the graph find approximately the square root of 20.
- (d) Extend the graph. From the extended graph find the square root of 105.
- (e) If y is 4.5, find the approximate value of the corresponding square x using the graph.

- 2. Verify the following 3 equations in the left column. Then fill in the blanks in the remaining three equations in the right column and verify.

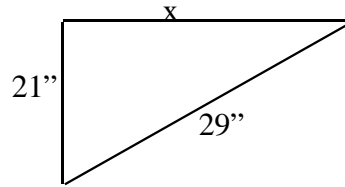
$5^2 = (5 + 2)(5 - 2) + 2^2$	$79^2 = (79 + 1)(\underline{\quad} - \underline{\quad}) + \underline{\quad}^2$
$7^2 = (7 + 3)(7 - 3) + 3^2$	$93^2 = (93 + \underline{\quad})(93 - \underline{\quad}) + \underline{\quad}^2$
$46^2 = (46 + 4)(46 - 4) + 4^2$	$115^2 = (115 + \underline{\quad})(\underline{\quad} - 5\underline{\quad}) + \underline{\quad}^2$

- 3. Find the value of x in the following right triangles using the Pythagorean Theorem.

(a)



(b)

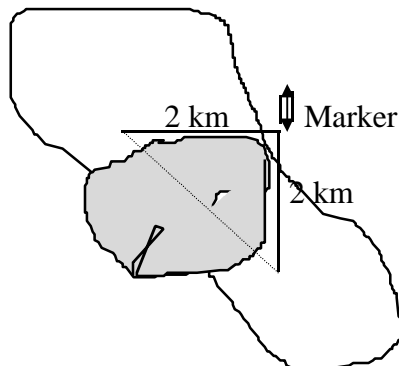


4. Complete the following table that represent the sides of a right triangle using Pythagorean Theorem. **c** is the hypotenuse.

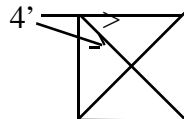
a	5	7	9		13
b		24		60	84
c	13		41	61	

5. An island is divided into two parts by a mountain. To build a tunnel through the mountain in a short time, the Department of Highways start digging from both sides. To make sure that they end up at the same point they devise the following plan.

They fix a marker in the ocean and at a distance of two kilometers on either side of the mountain and they start digging at 45° maintaining the same angle. How long is the tunnel?
 (Comment: Based on Heron's story. Source: Morris Kline, Mathematics and the Physical World, Page 95.)



6. A wooden frame is constructed using a spine of two strips measuring 4' connected in the middle at right angles and fixing four strips at the corners as shown. A sheet of canvas is stretched on the frame. What is the area of the stretched canvas?



7. Store A sells a 5 lb.-bag of corn flour (Masa Harina) for \$1.50. Store B sells a 2 Kg-bag of the same flour for \$1.20. Which flour is cheaper per pound? By how much? (1 kg. = 2.2 lb)

8. Canada uses the metric system. Shana began her trip from Toronto, Canada and drove at a rate of 90 km/hr for $2\frac{1}{2}$ hours. At Niagara Falls she entered the U.S. and drove at 65 mph for 8 hours to reach New York City. How many miles did Shana drive? What was her average speed in miles/hour.

Week 12

Concepts and skills: *Word problems involving fractions, equations, formulas, fractional equations, Ratio and Proportion, Denominate numbers*

1. A Beverage and Snacks Company has income from sales during 1995-1996 as in the following table. Assume that the income steadily increases from \$28,700 million to \$29,400 million during the year.

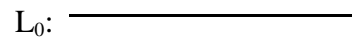
Date computed	7/1/1995	8/1/1995	1/1/1996	6/30/1996
\$ in Millions	28700			29400

- (a) What was the average monthly increase during 1995-1996? What was the income in the second month? What was the income in the month x?
 - (b) Draw a line graph using the above table.
 - (c) If the average monthly increase during 1996-97, what is the projected income on 8/1/1996? Answer the problem two ways: using ratio and proportion, and graph.
2. A bucket contains 2 gallons of 30%-solution of water and ammonia. 1 quart of 25% ammonia solution is added to the bucket.
- (a) Complete the following table.

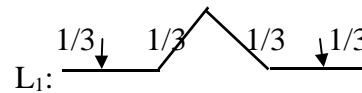
	Water	Ammonia	Total Volume
Original volume			8 quarts
Original percentage	%	30%	100%
Volume of ammonia solution to be added			1 quart
percentage	%	%	100%
Volume after adding ammonia solution i.e. New volume			9 quarts
New percentage	%	%	100%

3. (a) Find the length of each set of segments: L_1, L_2

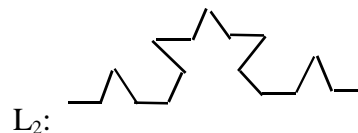
Step 0: Start with a segment, L_0 , of Length 1.



Step 1: Create scaled down copies of L_0 each having length $1/3$ of L_0 . Call the new figure L_1 .



Step 2: Create scaled down copies of L_1 each having length $1/3$ of a segment in L_1
What is the length of each segment?



(b) If this construction is continued, and L_3 is created, predict the length of L_3 . Is it greater than L_2 ?

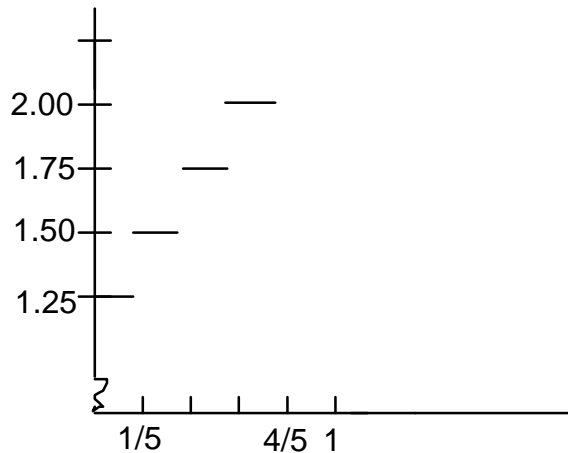
Notice that the length of L_3 is the greatest of the three sets. One can continue these steps. The curve we obtain is called the Koch Curve. Such curves can be found in nature like a jagged edge of an ocean beach. The length of such a curve becomes infinite.

(Source: COMAP, Lexington, MA 02173, Consortium Newsletter Number 45, Spring 1993, p. 6)

4. A cab fare is \$1.25 for the first $\frac{1}{5}$ mile and 25 cents per $\frac{1}{5}$ mile thereafter. Last month the cab fare was \$1.10 for the first $\frac{1}{8}$ mile and 15 cents per $\frac{1}{8}$ mile thereafter. By how much did a two mile ride go up?
 5. A cab ride costs \$1.25 for first $\frac{1}{5}$ mile and 25 cents per $\frac{1}{5}$ mile.
- (a) Complete the following table

Miles	0	$\frac{1}{5}$	$\frac{2}{5}$		$\frac{4}{5}$	1	2	3
Fare								

- (b) Draw a line graph using the table in (a). A part of the graph is already drawn .



- (c) If the cab fare for a trip was \$12.50, how many miles did the cab cover? (correct to $\frac{1}{5}$ mile.)
6. A departmental store has a store-wide sale. Lenny buys the items given in the following table.
- (a) Complete the table.

	Shirts	Trousers	Ties	Socks	Total
# items	2	2	4	5 pairs	xxxx
Price tag per item	23	29	7.50	3.50	xxxx
% discount	30%	40%	25%	25%	xxxx
Savings					
Sale price					

- (b) Find the total amount he has to pay.
- (c) How much is his saving?
- (d) How much would be the original total?
- (e) What percent on the entire purchase did he save?

7. A store gives an extra 10% discount if a charge account is opened. What is the actual percent discount if an item on sale is 40% off the lowest ticketed price, if you open a charge account at the store?
8. For every 115 pounds of newspaper that is recycled, a mature tree is saved. For 1,000 tons of newspaper recycled, how many mature trees are saved? (1 ton = 2000 pounds)
(Source: WAL*MART circular)
9. A distributor sells 200 copies of a book at \$8 per copy. She sells 100 copies for \$10 per book.
 - (a) Draw a line graph using the number of copies along x-axis and price per book on the y-axis.
 - (b) What is the ratio of change in price to change in the number of copies?
 - (c) From the graph find how much would a copy cost if she sells 300 copies at the same rate.
 - (d) Extend the line to intersect the y-axis. What is the meaning of the point of intersection?
(Note: Such a graph is called a demand curve in economics.)

Week 13

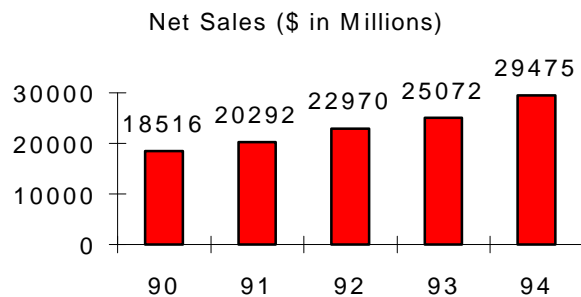
Review

1. A chef went on a diet. The first week the chef lost six pounds. During the five subsequent weeks, he lost two pounds each week. The chef weighed 250 pounds the day he went on a diet.
 - (a) Complete the following table.

Week	1	2	3	4	5	6
Total Pounds Lost						
Chef's weight						

- (b) Draw the graph of total pounds lost against weeks.
- (c) Draw the graph of Chef's weight against weeks.

2. A company's net sales from Jan. 1, 1990 - Dec. 31, 1994 are described by the following chart.



Using the chart answer the following questions.

- (a) What was the increase in sales from Dec. 31, 1993- Dec. 31, 1994. Find the percent increase.
- (b) If the percent increase in the year 1995 was the same as the previous year, what were the sales during the year 1995?

3. Match with each table in (a) through (d), the appropriate graph in Ga - Gd.

(a)

x	0	2	3		x
y	0	4	6	8	$2x$

(b)

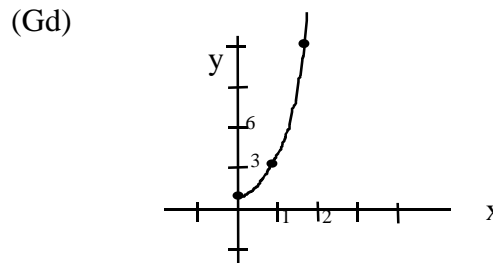
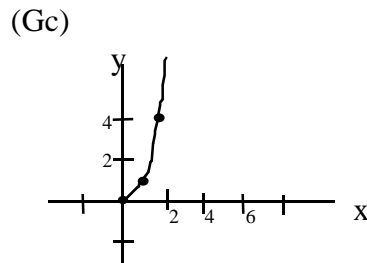
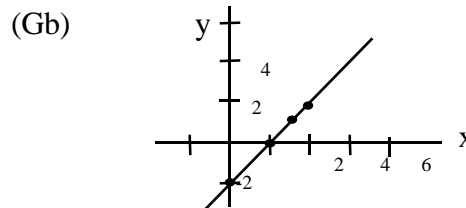
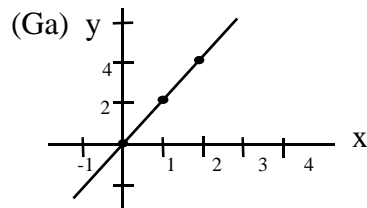
x	0	1		4	x
y	0	1	9	16	x^2

(c)

x	0	1	2	3	x
y	1	3	12	24	$3(2)^x$

(d)

x	0	2	3	4	x
y	-2	0	1	2	$x - 2$



4. Ruben invests \$5,000 in two types of stock. He invests \$2,000 in a growth stock and the rest in technology stock. The growth stock went up by 18% and technology stock went down by 5%. How much is Ruben's investment now? (Hint: Completing the following table will be helpful.)

	Growth	Technology	
Initial Investment	2,000		
Rate			
Change (\$)			Total Investment ↓
Current value (\$)			

5. A 1-lb bottle of mixed nuts contains 1/2 lb. peanuts, 1/3 lb. cashews, and remaining part other nuts. At the same rate, how many pounds of nuts other than peanuts and cashews are contained in a case of 24 such bottles.

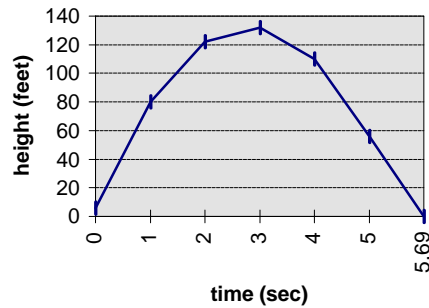
6. 1 gallon = 3.78 L, how many mL is 1 quart? (Use: 1 L = 1000 mL)

7. A salesperson makes \$475 and 10% commission for sales over \$500. The following table describes her salary. Find her salary if she sells merchandise worth x dollars.

Sales (\$)	0 - 500	1000	3000	x
Salary (\$)	475	$475 + .10(1000-500)$	$475 + .10(3000 - 500)$	

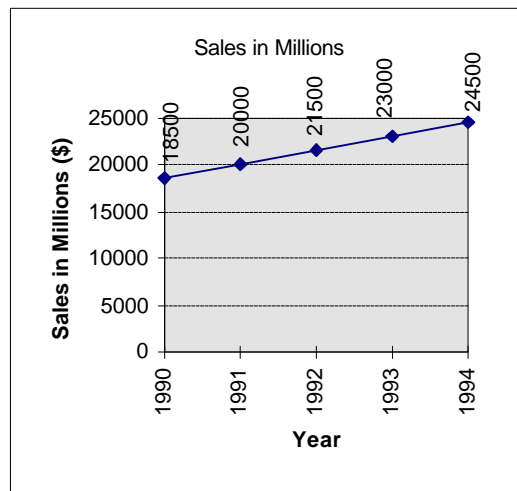
8. The following graph describes time and the corresponding distance from the ground of an orange tossed in the air.

Graph of a tossed orange



- (a) At what time does the orange reach the height of 90 feet?
- (b) How high does the orange reach and at what time?

The next two questions are based on the chart given which describes the net sales of a company.



- 9. What was the average increase per year from 1990 - 1994?
 - A. \$6000 million/year
 - B. \$3000 million/year
 - C. \$4500 million/year
 - D. \$1500 million/year
 - E. cannot be determined.

10. Based on the average yearly sales for the five years, what are the projected net sales for the year 1995?

A. \$1500 million

B. \$26000 million

C. \$23000 million

D. \$25000 million

E. \$27000 million

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Appendix 1 AMATYC STANDARDS

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GUIDELINES FOR CONTENT	
Increased Attention	Decreased Attention
pattern recognition, drawing inferences*	rote application of formulas
number sense, mental arithmetic, and estimation*	arithmetic drill exercises, routine operations in whole and real numbers
connections between mathematics and other disciplines	presentation of mathematics as an abstract entity
integration of topics throughout the curriculum*	algebra, trigonometry, analytic geometry, etc., as separate courses
discovery of geometrical relationships through the use of models, technology, and manipulatives*	establishing geometric relationships solely through formal proofs
visual representation of probability as area under a curve and via probability trees; of timelines for annuities and interest; of logic and electrical circuits	rote memorization and use of probability formulas
integration of the concept of function across topics within and among courses	separate and unconnected units on linear, quadratic, polynomial, radical, exponential, and logarithmic functions
analysis of the general behavior of a variety of functions in order to check the reasonableness of graphs produced by graphing utilities	paper-and-pencil evaluation of functions and hand-drawn graphs based on plotting points
connection of functional behavior (such as where a function increases, decreases, achieves a maximum and/or minimum, or changes concavity) to the situation modeled by the function	emphasis on the manipulation of complicated radical expressions, factoring, rational expressions, logarithms, and exponents with little obvious relevance to functional behavior
connections among a problem situation, its model as a function in symbolic form, and the graph of that function*	"cookbook" problem solving without connections
modeling problems with experimental and theoretical probability, including estimations based on simulations*	theoretical development of probability theorems

* Points applicable to Prealgebra

GUIDELINES FOR CONTENT (continued)	
Increased Attention	Decreased Attention
collection of real data for use in both descriptive and inferential statistical techniques*	analysis of contrived or trivial data
exploratory graphical analysis as part of inferential procedures*	"cookbook" approaches to applying statistical computations and tests which fail to focus on the logic behind the processes
use of curve fitting-to model real data, including transformation of data when needed	reliance on out-of-context functions that are overly simplistic
discussion of the fact that nonzero correlation does not imply one variable causes another	blind acceptance of r
use of statistical software and graphing calculators	paper-and-pencil calculations and four function calculators
problems related to the ordinary lives of students, e.g., financing items that students can afford and statistics related to sports participated in by females as well as by males*	problems unrelated to the daily lives of most students, e.g., investments of large sums of money in savings or statistics related to sports only played by males
matrices to organize and analyze information from a wide variety of settings	requiring a system of equations to be solved by three methods
graph theory and algorithms as a means of solving problems*	algebraically derived exact answers

* Points applicable to Prealgebra

GUIDELINES FOR PEDAGOGY	
Increased Use	Decreased Use
active involvement of students	passive listening
technology to aid in concept development	paper-and-pencil drill
problem solving and multi-step problems	one-step single answer problems
mathematical reasoning	memorization of facts and procedures
conceptual understanding	rote manipulation
realistic problems encountered by adults	contrived exercises
an integrated curriculum with ideas developed in context	isolated topic approach
multiple approaches to problem solving	requiring a particular method for solving a problem
diverse and frequent assessment both in class and outside of class	tests and a final exam as the sole assessment
open-ended problems	problems with only one possible answer
oral and written communication to explain solutions	requiring only short, numerical answers, or multiple choice responses
variety of teaching strategies	lecturing

Summary

These standards provide a new vision for introductory college mathematics—a vision whereby students develop intellectually by learning central mathematical concepts in settings that employ a rich variety of instructional strategies. To provide a more concrete illustration of these standards the Appendix contains a set of problems that brings them to life.

Appendix 2
Detailed Syllabus for MTH 002 - PREALGEBRA

Week	Topic/s
1	Whole Numbers: The Place Value System of Whole Numbers, Arithmetic of Whole Numbers, Distributive Property. Bar and Line graphs Problems for Classwork 1, Lab 1
2	Fractions (Brief Review); Decimals: Place Value System, Comparison of Decimals, Rounding and Estimation, Addition and Subtraction of Decimals Multiplication and Division of Decimals, Linear Measurement Problems for Classwork 2, Lab 2
3	Definition of Percent, Changing Percent to Decimals or Fractions, Changing a Fraction or Decimal to a Percent Finding a given percent of a given number, and finding percent when a part and whole are given, without using equations. Selected Applications of Percent, Word Problems Problems for Classwork 3, Lab 3
4	Exponents, Order of Operations, Variable Expressions, Geometric Formulas, Perimeter, Area, Volume Problems for Classwork 4, Lab 4 Test 1
5	Introduction to Signed Numbers, Addition, Subtraction and Multiplication of Signed Numbers Introduction to graphing - Instructor's Notes Problems for Classwork 5, Lab 5
6	Division of Signed Numbers, Combining Like Terms, Laws of Exponents Problems for Classwork 6, Lab 6
7	Addition Property of Equality, Multiplication Property of Equality, Equations combining both properties, Formulas and Word Problems Problems for Classwork 7, Lab 7 Test 2
8	Word Problems using equations, Transposition (Optional), Word Problems on Percent using equations, Interest formula Problems for Classwork 8, Lab 8
9, 10	Fractions (Rational Numbers), Graphing Rational Numbers, Rational Expressions, Reducing, Mixed Numbers and Improper Fractions, Operations on Fractions and Rational Expressions, Comparison of Fractions, Operations on Mixed Numbers Problems for Classwork 9, Lab 9, Classwork 10, Lab 10 Test 3
11	Fractional Equations, Cross Multiplication, Ratio and Rates, Proportion, Miscellaneous Applications of Percent - Mixture Problems Problems for Classwork 11, Lab 11
12	Square roots, Pythagorean Theorem, Denominate Numbers Problems for Classwork 12, Lab 12 Test 4
13	Word Problems - Review Problems for Classwork 13, Lab 13
	Final Examination - comprehensive