Research results from the College Preparatory Mathematics High School Program

A presentation to the
Fifth Conference on the
Teaching of Mathematics

Baltimore, Maryland
June 21, 1996

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Brief history of CPM *(College Preparatory Mathematics: Change From Within*) CPM is a high-school reform mathematics program conceived independently but built around the same principles as the Harvard Calculus Project. The texts currently available are Algebra 1, Geometry and Algebra 2, but a Pre-calculus book is being written this summer. The project was originally funded by an Eisenhower grant in 1989. Since then, it has approximately doubled in size every year to about 2300 teachers and 250,000 students today, mostly in California. Before teaching CPM, every teacher is required to attend eight days of inservice which is done at 32 sites by 140 teacher-leaders.

Education principles of CPM

Students should be actively involved in their learning.
Ideas need to be seen in many forms for understanding.
Teach fewer topics better.
Big ideas take a long time to learn.

Active involvement of students implies:

Changes in text;
Changes in assessment;
Change in student attitudes, beliefs and behaviors;
Change in teacher attitudes, beliefs and behaviors.

To understand a new mathematical idea, students need to:

Explore concrete situations numerically;
Represent related general situations algebraically;
Understand the associated geometric and graphical representations;
Be able to speak and write about their work.

An effective mathematics course needs to

be very selective about the ideas which are taught (so it should focus on the important ideas);
spend months investigating these important ideas in a spiraled fashion.

It is important to impart the following student attitudes, beliefs and behaviors

I am responsible for my own learning.
Math makes sense.
If I try, I can understand.
I can figure out most new ideas on my own.
To learn, I need to work.

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Seven studies about CPM which look at the performances of students on a variety of measures are summarized briefly below. Additional details of four reports are then given in some detail, including the complete sets of questions for the 1992 Algebra and Geometry studies.

- A study of three eighth-grade honors algebra classes showed that before CPM was introduced, about 50% of these students were taking calculus four years later, while after CPM was introduced, about 80% of the students were taking calculus.

- MDTP 1993 Assessments. The MDTP scores of approximately 4200 students at five schools who used the Math Diagnostic Testing Program multiple-choice examinations showed that CPM students scored about 20% higher on the algebra examination and about 10% higher on the geometry examinations compared to non-CPM students at the same school.

- CPM 1992 Algebra Study. 1740 algebra students (58% of them CPM) participated in an assessment where each student had 20 minutes to solve two written-response items. The results favored CPM students for both genders and all ethnic groups and were significant at the .0001 level. Overall, CPM students scored about 44% higher on the average test item.

- CPM 1992 Geometry Study. 608 geometry students (68% of them CPM) participated in an assessment where each student had 20 minutes to solve two written-response items. The results favored CPM students for both genders and all ethnic groups and were significant at the .0001 level. Overall, CPM students scored about 41% higher on the average test item.

- CPM 1993 Algebra Study-Northern California. 1795 students (53% CPM) participated in an assessment where each student had 30 minutes to solve three written-response items, one of which was either from IMP or a national examination. The results favored CPM students for both genders and all ethnic groups and were significant at the .03 level.

- CPM 1993 Algebra Study-Southern California. 860 students (52% CPM) participated in an assessment where each student had 30 minutes to solve three written-response items, one of which was either from IMP or a national examination. The results favored CPM students for

both genders and all ethnic groups and were significant at the .0001 level.

- CPM 1993 Geometry Study. 984 students (54% CPM) participated in an assessment where each student had 30 minutes to solve three written-response items, one of which was either from IMP or a national examination. The results favored CPM students for both genders and all ethnic groups and were significant at the .03 level.
- The California State Department of Education each year administers in algebra and geometry classes the "Golden State Examinations" a combination of multiple-choice and written response questions. The three levels of achievement for students are "school recognition," "honors," and "high honors." It is not uncommon for CPM schools to have three times as many honored students as students at non-CPM schools within the same district.

Straws in the wind. (Information which should be considered promising, but not definitive at this time.)

- Preliminary results from five high schools indicate that after introduction of CPM the SAT1 scores increase slightly (5-20 points) and the SAT2 (Math 2) scores increase by 10-80 points.
- More students of color appear to continue with CPM into geometry and algebra 2 and more females appear to continue into algebra 2 compared to students in traditional courses.

Each of the above questions is being studied at this time. Next year we hope to study the performance of former CPM students in first-year calculus classes.

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A Study of High-ability Students in CPM Classes.

Most of our early studies focused on the achievement of low-ability students in CPM classes. This is the first study to focus exclusively on very high-ability CPM students. To investigate the achievement of this group, we looked for schools in California which have been using CPM in classes designated as honors algebra or honors geometry for at least four years. There were only four such classes in the state and only three responded with data.

For these three, the results are quite dramatic. When schools switched either their honors algebra 1 or their honors geometry class to CPM, the percentage of students who took calculus four or three years later went up **59%** (from 51% before CPM to 81% after) in one case and **63%** (from 49% to 80%) in the other. In the third case, the high school taught no calculus until four years after CPM had been introduced in an eighth grade honors class. Now 22 out of 24 of those students (**92%**) take calculus. In all three schools the same teacher has taught the honors class (before CPM and then after CPM was introduced) and, in the two schools where a calculus course existed, the same teacher has taught the calculus class for the entire period of the study.

In the first school, the practice is now to take the best students from CPM Math 3 (Algebra 2) and put them directly into calculus. These students have all earned an "A" so far.

School A

Year of graduation	Type of program	Original number in class	% graduating from school	% of graduates taking calculus	Average SAT scores
1994	Non-CPM	47	98%	50%	569
1995	Non-CPM	55	93%	37%	543
1996	Non-CPM	19	89%	59%	561
1996	CPM	31	81%	80%	628

School B

Year of graduation	Type of program	Original number in class	% graduating from school	% of graduates taking calculus
1994	Non-CPM	38	87%	55%
1995	Non-CPM	60	85%	55%
1996	Non-CPM	35	86%	43%
1996	CPM	52	83%	81%

A Study of MDTP Results

The Math Diagnostic Testing Program is a well-established program which produces multiple-choice examinations for use by California high schools to provide diagnostic data for students and teachers and by colleges as placement tests into mathematics courses. The examinations are written and validated by a team of University of California and California State University faculty together with high school teachers. As such, their examinations were pressed into service as a benchmark that CPM students knew the basic computational skills of algebra and geometry.

In the summer of 1993, MDTP provided us with data from every school in northern California which could found for which 1/4 to 3/4 of the teachers who gave the examination were CPM teachers. This data was provided without names of either the teachers or the students, but with the classes identified as CPM or non-CPM. The results that follow are based on that information of about 4200 students in 91 classes of 34 different teachers at five schools. The schools are fairly typical urban/suburban schools which are lower to middle class. Three of them (A, C, and D) are trying to teach algebra to all students with all of the problems attendant to this goal--unmotivated students, etc. So the scores should be read with this background in mind.

The principal result is that **CPM students know the basic math skills better than students in traditional classes.** CPM students scored about 9% higher (geometry) to 20% higher (algebra) than non-CPM students at the same school. The scores are lower than one would wish, but given the clientele whose abilities are at the lower end of the ability range, the differences are important. These results should be read in conjunction with the previous study which looked at the performance of very high-ability students in CPM classes.

In the tables below, the numbers given are the number correct. The two algebra examinations each had 45 questions to be answered in 45 minutes, while the geometry examination had 50 questions to be answered in 45 minutes.

Geometry Test (SR45a93)

Schools	A	B	C	D	Average
CPM mean	20.61	21.14	18.74	19.78	20.07
Non-CPM mean	18.75	20.22	19.05	15.55	18.40
%CPM higher	9.9%	4.5%	-1.6%	27.2%	9.1%

Algebra Test (GR45a93)

Schools	A	B	Average
CPM mean	17.08	22.60	19.84
Non-CPM mean	16.11	15.81	15.96
%CPM higher	6.0%	42.9%	24.3%

Algebra Test (EA50a90)

Schools	D	E	Average
CPM mean	18.01	25.85	21.93
Non-CPM mean	15.16	22.23	18.70
%CPM higher	18.8%	16.3%	17.3%

College Prep Math Assessment in Algebra I and Geometry 1992 Results

Executive Summary

This report is a careful look at the results of a study of approximately 2400 students from Algebra and Geometry classes who participated in an assessment during May of 1992. Each student worked on two of the exam questions shown in Attachment 1 and was required to show his or her work. A typical question was:

. The city of Nampa currently has 45,000 people and is growing at 13,000 people per year. Caldwell has 66,000 people and is growing at 9,000 people per year. In how many years will the two towns have the same population?

Papers from CPM and non-CPM students were coded so that the students could not be identified and mixed together to guarantee uniform grading. Each paper was graded holistically on a scale of 0-4 with 3 representing satisfactory work and 4 representing excellent work.

The overall result is that CPM students have uniformly higher means on these tests than students from non-CPM classes. The results are true for both sexes, all ethnic groups, all grade levels and are significant overall at the .0001 level; that is, there is a probability of .0001 or less that the CPM and non-CPM students performed equally well. Equally good results appear when we compare the percentages of CPM and non-CPM students who score a 3 or 4 on given questions.

Methodology

In May, 1992, examinations were sent to CPM teachers at 19 schools. Each CPM teacher was asked to find non-CPM teachers at his or her school or at a comparable school who would give the examination to their students. (Here we use the term "non-CPM" or “traditional”to denote a class using any one of the various standard textbook series--e.g. Haughton-Mifflin, etc.) The goal was to be assured of roughly comparable groups of students taking the examination. Twelve of the schools had both non-CPM and CPM students taking the examination, students at five CPM schools were paired with non-CPM students at other schools and two CPM schools were unpaired. For the algebra examination, 21 CPM teachers were involved, 14 non-CPM teachers and 4 teachers who taught both a CPM and non-CPM classes. Three IMP teachers also gave the algebra examination to their students, but they are excluded from this report. There were 8 CPM geometry teachers and 5 non-CPM geometry teachers. All of the teachers involved were volunteers.

Each of the students was asked to fill out a brief questionnaire detailing their age, grade level, gender and ethnicity. Then students were given 20 minutes to work on the two questions from one of six forms of the assessment. (The questions are shown in Attachment A.) Care was taken to ensure that each of the tested classes used approximately the same number of each of the six different forms of the test. Each teacher also filled out a questionnaire giving their gender, number of years of teaching experience, the text used, and the number of years they had taught from this particular text.

All of the examinations were sent to the CRESS Center at UC Davis, where each student's responses were coded by the same number on their information sheet and each of their two response sheets. Then the papers were separated into piles by question number and the papers within each pile were shuffled so that they would be graded in random order. Papers were returned from 1830 students who took the Math 1 (Algebra) assessment and 664 students who took the Math 2 (Geometry) assessment. From these original numbers, we were able to salvage 1740 Math 1 and 608 Math 2 complete data sets. CPM students comprised 1001 (57.5%) of the algebra responses and 414 (68.1%) of the geometry responses.

During June of 1992, a group of 30 teachers participated in the grading which was done holistically on a 5-point scale. On this scale a 0 represented no useful work (either nothing done or nonsense); 1 represented an understanding of the problem, but little progress toward a solution; 2 a reasonable start toward a solution, but an approach with significant gaps or misconceptions; 3 an almost complete solution which might contain minor errors; and 4 a full and complete solution. For each problem, the group agreed on a rubric for the scoring by looking at several sample papers. After the group had come to an agreement the shuffled papers were each graded by at least two people and, in case of disagreement, by a third person. The consensus score was recorded.

Analysis of the data.

All of the data was analyzed by the SPSS statistical package with the assistance of Neil Willett of the UC Davis Statistical Consulting Laboratory. In the first pass of the data it was determined that there was no significant difference in the performance of males and females on the same problem, but there were significant in performance among different ethnic groups, grade levels, teachers, schools and programs. Because of differences between individual teachers and schools, the data was analyzed by a *random effects* analysis, where we assume that differences between schools and teachers exist as well as by a *fixed effects* analysis, where we assume that all teachers and schools are substantially identical. We are, of course, most interested in comparing the CPM program with non-CPM program students.

In each case of program comparison, the difference favored CPM students, generally by a substantial and statistically significant margin.

When ethnic differences existed within a program, the white and Asian (and sometimes Hispanic) students scored higher than Blacks. However, CPM students of all ethnic groups outscored traditional students of all ethnic groups. That is, Black CPM students scored higher than students from any ethnic group in traditional classes. (This is illustrated in the graph of under the Executive Summary.)

In addition to comparing overall means of the two groups, we compared the proportion of those who scored a 3 or a 4 on their responses. These were the responses that were considered essentially correct. Again, the proportion of CPM students who scored a 3 or a 4 was significantly higher than the proportion for students in traditional classes.

Attachment A

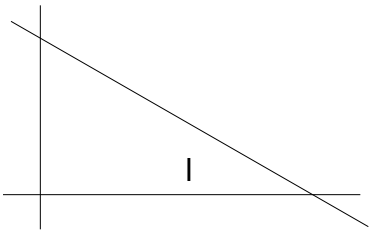
The questions follow with CPM and non-CPM mean scores (out of 4) in parentheses afterward. A single asterisk (*) means that the differences in scores were significant at the .05 level, and ** means that the scores were significant at the .01 level.

1992 Algebra I Assessment.

A1. A scientist is growing two experimental plants in his laboratory. Today, the small-leaf plant weighs 20 grams and is increasing in weight at 5 grams/day. The large-leaf plant weighs 11 grams and is increasing in weight at 7 grams/day. After how many days will the two plants together weigh 100 grams?

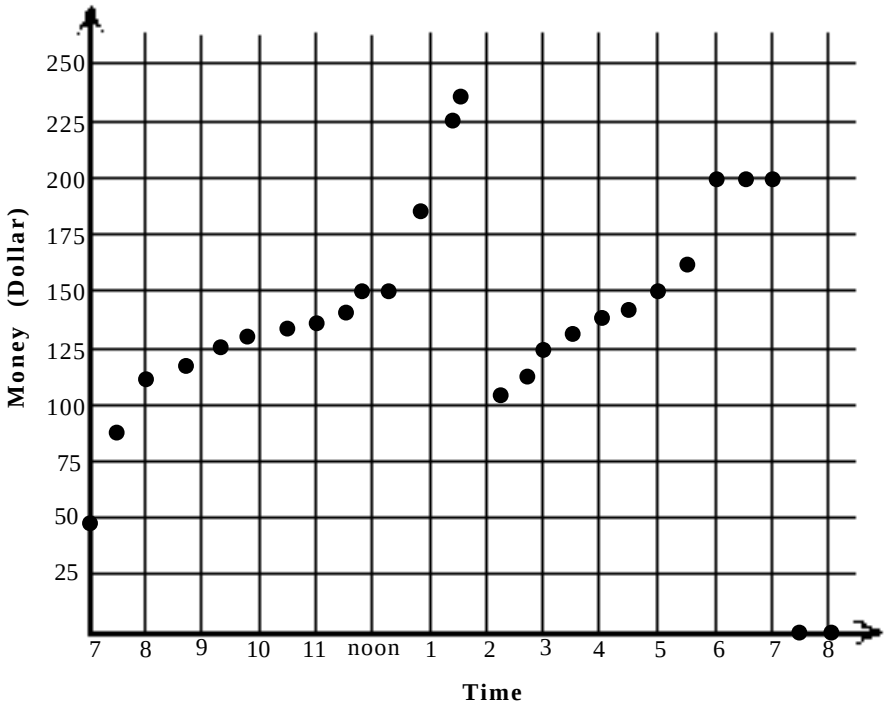
A complete response includes not only your work, but also an answer to the question, an appropriate equation or equations, and a statement about what each variable which you use represents. (2.53, 1.93) **

A2. Estimate the slope of the line L at right and explain how you got this value.



(2.03,1.26) **

B1. The graph below shows the amount of money in the cash register at Lincoln Cleaners during the day. Write a detailed description of what you think happened throughout the day at Lincoln Cleaners.


Money (Dollar)
250
225
200
175
150
125
100
75
50
25
7
8
9
10
11
noon
1
2
3
4
5
6
7
8
Time

(2.66,2.55)

B2. A rectangular blackboard has width which is 75% of its length and a perimeter of 148 centimeters. Find its dimensions to the nearest tenth of a centimeter. (2.02,1.39) *

C1. The city of Nampa currently has 45,000 people and is growing at 13,000 people per year. Caldwell has 66,000 people and is growing at 9,000 people per year. In how many years will the two towns have the same population?

A complete response includes not only your work, but also an answer to the question, an appropriate equation or equations, and a statement about what each variable which you use represents. (3.00,2.14)**

C2. A decorative window is in the shape of a square with a half circle on top of it. The area of the window is 30 square feet. Find the lengths of the straight sides of the window. (1.33,1.06)

D1. You are in charge of buying potato salad for the annual Central High picnic and are told that 50 pounds of salad will feed about 140 people. About how many pounds of salad should you buy if you want to feed 300 people?

A complete response includes not only your work, but also an answer to the question, an appropriate equation or equations, and a statement about what each variable which you use represents. (3.11,2.46) **

D2. Find as accurately as you can where the graphs $y = x^2 + x - 6$ and $y = x - 3$ intersect. Make sure you state both the x- and y-coordinates of the point or points of intersection.
(2.27,1.29)**

E1. Two knives weigh as much as three forks. Five forks and two knives weigh a total of 28 ounces. How many ounces does a fork weigh?

A complete response includes not only your work, but also an answer to the question, an appropriate equation or equations, and a statement about what each variable which you use represents. (2.14,1.99)

E2. Begin with a square ABCD with side length 12 cm and draw two quarter circles of radius 5 cm. inside the square, one centered at A and one at C. What percentage of the total area of the square is not inside one of the two circles? Be sure to show a diagram along with your work. (1.76,1.04) *

F1. You are given the polynomial $x^2 + 7x + \_\_$.

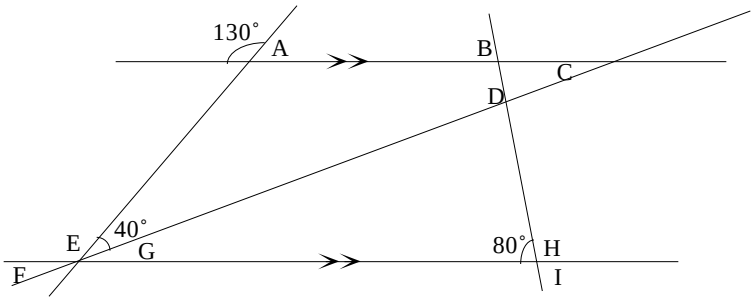
(a) Fill in the blank with some number so that one of the factors of this polynomial is
$(x + 4)$, and show the polynomial in factored form.

(b) Is it possible to fill in the blank with a different number than you used in (a) and still have (x+4) as a factor? Explain clearly why or why not. (2.81,2.12) **

F2. A rectangle is three times as long as it is wide. Find its dimensions to the nearest one tenth centimeter if its diagonal is 72 cm. Be sure to show a diagram along with your work.
(1.62, .78) **

Geometry Assessment 1992

A1. Find as many of the angles A, B, C, D, E, F, G, H, I as you can. Show how you found them.

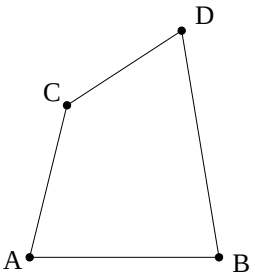


(2.32,2.02)

A2. A quadrilateral has vertices at A = (0,0), B = (10,0), C = (2,8) and D = (8,12).

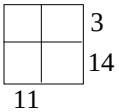
a) Find its area.

b) Find the measure of angle A.



(1.71,.97) *

B1. Fill in each of the squares with one of the integers 1, 2, …, 9. The sum of the two numbers in the top row is 3; the sum of the two numbers in the bottom row is 14 and the sum of the two numbers in the left column is 11. Find as many different solutions as you can and **explain clearly** how you know you have found them all.



(2.50,1.83) *

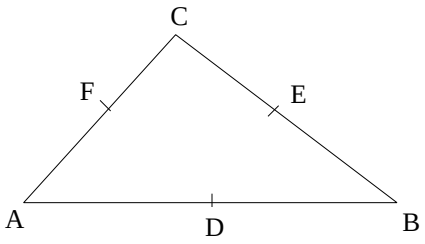
B2. A regular pentagon ABCDE has edges of length 2 cm. Find the length AC. (1.35,.55) *

C1. Begin with a regular 7-gon ABCDEFG. Extend sides AB and DE until they meet at P. Find the measures of each of the angles of the pentagon APEFG. (2.76,1.21) **

C2. A pyramid has four sides which are equilateral triangles of edge length 10 cm and a base which is a square. It weighs 200 grams. The top of the pyramid is cut off by a plane parallel to the base, so that you have a similar pyramid with edges 2 cm long. Find the weight of the small pyramid. (1.66, 1.50) *

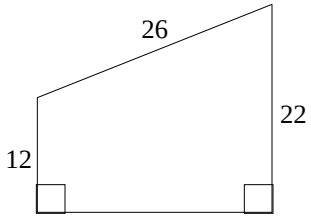
D1. A pyramid has a base which is an equilateral triangle and all of the other faces are congruent. Must each of the other faces be an equilateral triangle? Explain clearly why or why not. (2.00, 1.98)

D2. In ΔABC, AB = 30 cm, BC = 22 cm and AC = 18 cm. Point D is the midpoint of AB, E is the midpoint of BC and F is the midpoint of AC. Give a convincing argument or prove that ΔBDE is congruent to ΔEFC.



(2.28, 1.59) *

E1. Find the area and perimeter of the trapezoid at right. All lengths are in centimeters.



(3.19, 1.99) **

E2. a) Solve for x: $5(x - 1) + 7 = 15$

b) Solve for y: $y^2 - 5y + 6 = 0$

c) Solve for x and y: $2x - y = 7$
$y = x - 3$ (2.56, 2.55)

F1. A quadrilateral has vertices at A = (0,0), B = (4,0), C = (-2,8) and D = (16,8). Join midpoints of the sides of ABCD.

a) You get a special kind of quadrilateral when you do this. What kind?

b) Give a convincing argument or proof why your answer in part (a) is correct. (1.79, 1.93)

F2. A scientist is growing two experimental plants in his laboratory. Today, the small-leaf plant weighs 20 grams and is increasing in weight at 5 grams/day. The large-leaf plant weighs 11 grams and is increasing in weight at 7 grams/day. After how many days will the two plants together weight 100 grams?

A complete response includes not only your work, but also an answer to the question, an appropriate equation or equations, and a statement about what each variable which you use represents. (1.76, 2.20)

Below, we show the data collected from each student--both algebra and geometry.

Name: _____ Your grade level (9, 10, …): _____

Teacher: _____ Name of class: _____

School: _____ Textbook: _____