

## **How the Use of the TI-92 Affects Teaching Calculus**

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### **Introduction**

In the current mathematics reform, there is an emphasis on using technology, especially calculators, in the classroom (National Council of Teachers of Mathematics [NCTM], 1989, 1991, 1995). Texas Instruments [TI] has created an advanced computer algebras system (CAS) in a graphing calculator, the TI-92. The TI-92 combines the features of the graphing calculator with the symbolic manipulation capabilities of DERIVE CAS (Texas Instruments, 1995).

In the context of teaching mathematics, the question becomes how to integrate this tool into the teaching and learning of students of mathematics. This study looked at incorporating the TI-92 in teaching calculus. The focus was examining how students gained understanding of derivative and integral while learning with access to the TI-92.

The paper describes the course that was the focus of the study. It explains the theoretical perspective used in examining the teaching and learning of calculus in this context and how data was collected. Next, the paper gives a detailed explanation of how I, as instructor, addressed the topics of derivative and integral in the course. It also explains the knowledge that the students demonstrated to me. Finally, the interpretation of the findings and implications are given.

### **Setting and Population**

The course that was the focus of this study was Elementary Calculus I, calculus for non-majors, at the University of Maryland at College Park. The majority of students that

take this course are biology and business majors, who are taking the course as a major requirement. Biology majors and certain business concentrations must also take Elementary Calculus II. This section of the course was an experimental section redesigned for elementary education majors with a mathematics/science concentration. It was open, however, to all students. Twelve students were enrolled in the course: three elementary education majors, one journalism major, two biology majors, and the remainder were business majors. TI-92's were provided for all students.

The class followed the content of the textbook *Calculus Concepts: An Informal Approach to the Mathematics of Change* (LaTorre, Kenelly, Fetta, Harris, Carpenter, 1995). The book focuses on using mathematical models to study rates of change of real-world problems. It assumes that students have access to a graphing calculator or CAS.

The first two chapters deal with mathematical modeling using the regression features of the graphing calculator. The next three chapters deal with rates of change and derivatives. The next two chapters deal with sums of change and integration.

In the class, cooperative learning groups were used frequently, as well as whole class instruction. Very little in-class work was done individually. Students were assessed individually through homework and projects, journals, and examinations. They also completed a final portfolio containing 7-10 pieces of their best work from the course, covering the breadth of topics introduced in the course.

### **Theoretical Perspective**

The teaching style employed in the classroom was based upon constructivist theories of learning (Cobb, 1994; von Glasersfeld, 1990, 1996). In brief, students construct their own understandings from their own experiences. This construction is done through a process of reflective abstraction. Students reflect on their experiences and resolve conflicts or disequilibrium so that their previous understandings are abstracted, refined or rearranged to become viable with their understandings of current experiences. This reflection may be prompted by the individual student, the student's peers, or the

instructor. The role of the teacher is to facilitate this reflection. The teacher provides experiences that should invoke disequilibrium and help students to resolve this disequilibrium by encouraging and prompting reflection or allowing one another to prompt reflection.

In addition to the perspective that all students construct their own learning, a specific view of understanding mathematics was the basis of the instruction. First, students gain both a conceptual and procedural knowledge of mathematics (Hiebert, 1986). Secondly, students are enculturated into the community of practice of mathematicians, allowing the students to develop the habits of mind of mathematicians (Brown, Collins, & Duguid, 1989).

To elaborate on this view of the meaning of understanding in the context of calculus, I expected the students to conceptually understand what the derivative is. Conceptually, I wanted them to understand that the derivative is the slope of the tangent line, an instantaneous rate of change. Procedurally, I wanted the students to be able to calculate the derivative graphically, numerically and symbolically. Similarly, the students should understand that the integral is the area between the curve and the x-axis and the antiderivative. Procedurally, they should be able to calculate the integral, graphically, numerically, and symbolically. Finally, I wanted the students to recognize the value that mathematicians place on elegance in a solution process, and to seek elegant solutions. Primarily, in the context of calculus, I wanted them to use symbolic methods, where possible.

### **Data Collection and Analysis**

Class meetings were videotaped for several weeks, beginning with instruction on the derivative. My lesson plans for each class period are also part of the data set. I used this data to recreate the chronology of instruction. I collected all student homework and examinations on the topics of derivative and integral, which I used to determine their understanding. Students wrote weekly in a journal, which was sent to me via electronic

mail. In these journals, students were asked to write about specific mathematical topics, often prompted by questions in the book (LaTorre, et al., 1995). In addition, interviews were conducted with two students by another researcher with whom the students were familiar.

### **Chronology of Instruction**

In this course, the students were first presented with real world situations in which they determined average rates of change. They were then asked to explore rates of change over smaller intervals. I prompted them to consider the need for instantaneous rates of change by considering the situation of a car drive to a local beach. They were asked how a radar detector would be used to determine their speed at a particular moment. The students had already used ultrasonic motion detectors with the Calculator-Based Laboratory system (CBL), and had an idea of how the devices worked. They were able to see a need for a more refined method of calculating velocity over a shorter period of time. Thus, disequilibrium was achieved.

To resolve this disequilibrium, I prompted the students to reexamine the average rate of change. Recognizing that the average velocity is the slope of the secant line between two ordered pairs of a function, they were prompted to draw and calculate the slopes of the secant lines between a fixed point and points that progressively moved closer and closer to the fixed point from both a positive and negative direction. For a function  $f$  and a point  $a$ , the students were able to use the table feature of the TI-92 to examine the function  $[f(a+x) - f(a)]/x$ , for very small values of  $x$ . From the earlier work drawing and calculating the slopes of the secant lines, they noted that this function gives the slopes of the secant lines and is undefined at zero. They also recognized that as  $x$  gets closer to zero, from both the positive and negative directions, that the value is closer to the velocity at that instant.

Discussions of instantaneous velocity and slope the tangent line followed the discussions in the book. Students were told that the derivative is the slope of the tangent

line and were able to see this displayed on the TI-92, with the equation of the tangent line. They knew that the  $m$  value in the equation of the tangent line was the slope. Discussions followed of instantaneous rates of change of various functions at a single point. The students then used the CBL system with the TI-92 to explore the graphical relationship between a function and its derivative by noting where the distance was increasing and decreasing over time, and where the velocity was positive and negative over time. Then the concept of the slope formula was introduced as a more efficient means of finding the derivative for several points. They used the TI-92's symbolic manipulation capability, and found several derivatives of polynomial, power and exponential functions. They used the information to make generalizations about what the derivative rule must be for those types of functions and the effect of multiplying a function by a constant on the derivative of the resulting function. We discussed that to prove this they must use the four step method, but we did not do strict proofs.

Students were asked to use the four step method of finding  $f(x)$ , then  $f(x+h)$ , then  $[f(x+h)-f(x)]/h$ , then the limit as  $h$  goes to zero of  $[f(x+h)-f(x)]/h$  (LaTorre, et al., 1995). Because there was no formal treatment of limits, students used their calculators to plug in different values of  $h$ , as  $h$  moved closer to zero, and realized that for continuous functions  $[f(x+h)-f(x)]/h$ , that they could substitute zero for  $h$ . The students calculated the slope formulas using this method for several polynomial functions. They used the calculator to calculate the derivative at certain points and generalize to determine the slope formula of  $\ln(x)$  (LaTorre, et al., 1995). Hence, graphical, numerical and symbolic approaches were used to introduce and further the concept of derivative. The topic of more complex derivative rules followed.

From their work with polynomials, they realized that the derivative of a function that is the sum of two functions is the sum of the derivatives of those two functions. The chain rule was less apparent. By using the graphing capabilities of the TI-92, they realized that the "inner function" changed the "outer function" in a systematic fashion. Hence,

they were able to recognize how the chain rule works. The book presents several corollaries of the chain rule for special cases, which the students used to practice exercises and complete problems. Students were then given the product rule, and were able to practice exercises and complete problems using this rule.

Students did not rely on the symbolic manipulation capabilities of the TI-92 to calculate derivatives. The form in which the calculator simplified the answers was not the same as the answers that they received from using the derivative rules. Hence, I knew when they used the calculator to determine the derivative. Students only used the calculator to calculate 2<sup>nd</sup> derivatives of logistic functions or functions that required to product rule. They also only used the calculator to calculate derivatives in the context of problem situations.

After addressing applications of the derivative, such as optimization and estimation, the students were then exposed to the area under the curve as a way of summing change. Using a constant velocity curve, they recognized that  $d=v*t$  and  $v*t$  as the area of the rectangle formed by the axes, the curve, and the vertical line drawn at the time. They then did numerical explorations of the area under the curve by creating rectangular and trapezoidal sums using a program on their TI-92's. They recognized that the more rectangles/trapezoids used the better the approximation, when the actual area was given. They also recognized that midpoint approximations were the best. Simpson's rule, a method of weighted averages between the midpoint and trapezoidal approximations, was discussed.

They were then introduced to the process of limiting sums by progressively using more rectangles, and that the limit as this number of rectangles approaches infinite is the integral or area between the curve and the x-axis. They were then introduced to integral notation. Students then used the integral features on the TI-92 to compute the definite integral of several similar functions to generalize a rule for finding the indefinite integral. This can also be done on the less sophisticated TI calculators. These rules were related

back to the graphical representation. This section followed activities provided in the book.

Finally, students were asked to reflect on all of the relationships that they knew existed between distance and velocity. From there, the class discussed the fundamental theorem of calculus and the concept of antiderivative was introduced. Antiderivative rules were given and related to derivative rules by having the students find the antiderivative, then the derivative of the result. They used this information to then calculate definite integrals using the antiderivative rules. We also covered the sum rule by exploring the graphical representation. This led to a discussion of the area between curves. Finally, we discussed the properties of definite integrals by exploring the graphical representations.

### **Students' Knowledge**

The students conceptually understood both derivative and integral very well. When given a problem in which it was necessary to calculate the instantaneous rate of change, or area under the curve, all students were able to recognize the need, and correctly compute the derivative or the integral. Procedurally, all but one student, who had infrequent attendance, were able to compute the derivative and integral graphically, numerically, and symbolically, on demand. The evidence for this claim is in their answers to test questions asking them to find the slope formula or indefinite integral. When asked how they would find the derivative at a given point if only given a graph, they all responded that they would find the slope of the tangent line. They were able to quote which derivative rules that they used in computing the slope formulas. Similarly, they were able to quote the antiderivative rules that they used in computing the indefinite integral.

However, unless a slope formula was asked for explicitly, half of the students resorted to graphical methods to solve the problems. This generally entailed graphing the function with the TI-92, then using the tangent function to find the tangent line, and using the slope obtained. Rarely did students use the symbolic manipulation characteristics of

the TI-92 to calculate the slope formula. Only in lengthy, multiple chain rule or product rule types of problems did they do this. The TI-92 was used to solve algebraic problems, such as solving the derivative for zero in optimization problems. Numerical methods were rarely used unless given data that did not fit any model particularly well.

Similarly, unless students were asked to use the fundamental theorem of calculus to find the definite integral, all of them resorted to rectangular approximations using the TI-92 program provided for them. If asked how to make their answer more accurate, they would respond “Use more rectangles” or “Use Simpson’s rule” not “Use the fundamental theorem.” Often, two students would approximate the area with an increasing number of rectangles and use their naive concept of limit to find a limiting value as the integral. The remainder would use a large number, greater than 100, and use or round this result as the integral.

### **Interpretation and Implications**

The students gained both conceptual and procedural understandings of derivative and integral. However, for many, when given a choice, they were more likely to use their conceptual understanding to solve a problem. This finding is remarkable considering that the TI-92 is able to do many of the procedures. Thus, the concern that the students will rely on the CAS, and not learn the procedures did not materialize. However, I have several hypotheses that may explain the fact that the students relied upon conceptual understanding to solve problems.

First, as documented in the chronology of events, I presented students with the opportunity to learn the concepts of derivative and integral prior to learning procedures for finding the derivative or integral. Also, I presented the procedures beginning with more conceptually oriented procedures, such as graphical methods, then numerical methods, and finally symbolic methods. Hence, they may have developed more facility in the conceptually oriented procedures because they had more time to practice them. Also, research on children’s addition strategies shows that students do not always move on to

use a more sophisticated strategy that they may understand, until they are fully comfortable and confident that the strategy always works (Carpenter & Moser, 1984). This explanation may also apply to these college students.

Secondly, schema theory may be used to explain the students' lack of use of symbolic procedures. Briefly, schema theory is a theory of learning that posits that the brain stores information "as a mental representation of what all instances of something have in common" (Byrnes, 1996, p. 19). They serve the function of categorizing information, helping to remember and comprehend information, and adding to problems solving abilities. This theory is consistent with constructivism, in that students gain schemata from experiences, and refine schemata as needed, through abstraction (Byrnes, 1996).

In the case of these calculus students, they may not have had enough experience linking the more conceptual procedures to the symbolic procedures to refine their schema to include symbolic procedures in their problem solving strategies. Also, because symbolic procedures do not always work, they may have refined their schema to only include them in a solution strategy if the problem specifically requires a slope formula or indefinite integral.

Thirdly, I, as instructor, may not have conveyed the importance of more refined methods or which of the methods studied are more refined in the enculturation process. By attempting to have students act as a community of mathematicians, I may not have conveyed the mathematician's premium placed upon elegance. In calculus, a symbolic solution is often considered by mathematicians to be more elegant than a graphical solution. In assessing students during the course, I did not place a greater value on more elegant solutions. Students place importance on what they are assessed (NCTM, 1995). Because students were interacting with real-world problems, accuracy was determined by the problem situation and the students and I determined that the answer should be

accurate in terms of the context of the problem. This often meant that graphical and symbolic solutions would produce the same answer for that context.

I chose not to demand a specific solution process most of the time because I felt that such a demand is counter to the spirit of reforming mathematics instruction and genuine problem solving (NCTM, 1989, 1991). I encouraged multiple solutions, but I did not place a higher value upon symbolic solutions. Hence, in the situation of their learning, they may not have been properly enculturated into the community of mathematicians.

Finally, I may have skewed the enculturation process by focusing on efficiency instead of elegance. In the past, an elegant solution tended to be a more efficient solution, such as a shorter proof or fewer steps. For example, to find the derivative, a graphical solution requires drawing the graph of the function, drawing the tangent line at a specific point, and calculating the slope of that tangent line. The first two steps require manual dexterity and time if they are to be done properly and accurately, by hand.

However, with the use of the TI-92, these steps become trivial. Hence, using the TI-92 made previously inefficient, inaccurate and time-consuming methods, such as, drawing 100 rectangles, calculating their areas and summing these, become both efficient and accurate. Thus, the students may have placed a premium on efficiency, but determined the use of the TI-92 is very efficient. They may not have seen a need to move to a traditionally more efficient method, symbolic manipulation, because their method worked all of the time and even on a less sophisticated graphing calculator.

My method to move students toward symbolic manipulation was to ask them for the derivative at several points. Finding the slope formula, to me, seemed to be the most effective way of doing this. However, using the TI-92's graphical capabilities, the students were able to find the derivative at several points and remove the human error that may come in computation. Graphical methods are also conceptually closer to the meaning of derivative or integral. Therefore, students must move to a more abstract level of thinking, which I may not have properly encouraged as the instructor.

## **Final Thoughts**

There is much more research to be done on the use of graphing calculators in the teaching of mathematics. I have listed several reasons that may explain why my students have gained the particular understanding that they did, but other explanations may exist. There are also ways to combat each of the problems I noted. I could have been more explicit about the premium placed upon symbolic methods. I could have asked students to use symbolic methods wherever possible, and lowered their scores when they did not. I could have forced students not to use the TI-92's during specific periods of time. Perhaps, for those taking another calculus course, with more practice they will become comfortable with moving to more abstract methods.

There is another perspective that can be taken. Enculturation into the community of mathematicians may not mean that the students have to take on all of the values of mathematicians, such as the value placed on elegance. Or, with the advent of technology and its use in solving mathematical problems, perhaps mathematicians may value efficiency due to technology as much as efficiency due to elegance. Elegance may take on a new meaning that includes the use of computational tools. Mathematics is socially constructed, and perhaps the premium placed upon elegance over simply finding the solution may no longer be viable in this situation (Cobb, 1994). When the TI-92 clashes with the community's beliefs, disequilibrium may follow, leading to a process of reflection. Upon reflection, these graphical methods may be seen to be as elegant as the symbolic methods. I encourage all members of the community to reflect upon this concept to resolve the disequilibrium.

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