

## Using Physics to Teach Calculus in an Integrated Course

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In calculus the derivative measures the rate of change and the integral computes the limit of the sum of incremental parts so that there are many applications in physics. For example, the derivative is velocity, the rate of change of distance with respect to time, and the integral is work, the limit of the sum of the product of force and increments of distance. To individuals who have a thorough understanding of the definitions of these two fundamental concepts of calculus, the above connections are clear. To students struggling with these definitions, the connections are often clouded.

In an integrated calculus/physics course such as that taught at Texas Woman's University as a part of the Foundation Coalition Engineering Education program, there is an opportunity to use applications of calculus in physics to reinforce (or perhaps to reteach) calculus concepts.

Access to reform calculus materials such as the Calculus Consortium at Harvard text support these connections through the representation of functions as data sets and graphs as well as in analytical form. This gives students an opportunity to experience

derivatives and anti-derivatives as something other than equations generated from given equations. The derivative of a function having a graphical representation is easily interpreted as the limit of the slope of the tangent and hence the slope of the curve. The derivative of a function having a numerical representation lets students see how the average rate of change becomes the instantaneous rate of change as limits are evaluated. The integral of a function pictured graphically motivates the relationship between area under a curve and the value of the integral. In numerical form the Riemann sums strongly motivate the definition of the integral. Since differentiation and anti-differentiation are introduced almost together, easy differential equations found early in physics can be "solved" in an intuitive way.

An example of a physics concept that can be used either as a teacher presented topic or as a team assignment to be done in class is the derivation of the formula for the moment of inertia of some solid of uniform density about an appropriate axis. The teacher can select a solid, choose a subdivision of  $n$  equal parts, determine the moment of inertia of each part, sum the partial moments and somehow take a limit to complete the derivation. Since the emphasis is on the concept of integration this process is useful, although somewhat tedious. Throughout the computations one question should be repeatedly asked of the class. "Is there an easier way to do this?" Answers might include: "Use DERIVE (or some other computer algebra system or a graphing calculator) to work out the summations and/or take the limit." "That's an

integral. Use the Fundamental Theorem to evaluate it." "That's an integral. Use DERIVE to evaluate it." Since the desired calculus concept is the definition of the integral, either of the last two answers is best.

The best use of an example of this sort is to give each of your classroom teams a different solid to work with and hope that at least one of them will recognize the definition of a definite integral.

The end result of activities such as this one should be a better understanding of what an integral is and improvement in the problem solving skills of the class members.