Brief *Calculus* - Not *Algebra*

Gene W. Bennett
University of Evansville

Use of DERIVE in a one-term calculus course facilitates the teaching of calculus concepts without being unduly impeded by student deficiencies in algebraic manipulative skills.

I. The Course: Brief Calculus for General Education And/Or Service

Many schools offer a calculus course that may be entitled *Brief Calculus*, or *Survey of Calculus,* or *Topics in Calculus.* The course is usually for non-math, non-natural science majors, and may be intended to serve as a general education course and/or a course for targeted majors in business, psychology, nursing, etc. The specific topics included in the course may vary from school to school, but there are some objectives common to most such courses:

1. Students are to learn what calculus is all about. That is, they should become acquainted with the major concepts, definitions, and theorems relating to limits, continuity, the derivative, and the definite integral.

2. Students should come to some appreciation of the applications of calculus to their respective areas of study. They should develop some sense of the types of problems to which calculus is appropriately applied.

3. Students should be able to ***do*** some calculus. They should become able to formulate a problem in appropriate terminology and notation, and to use principles and techniques learned in the course to solve the problem.

II. The Problem: Students' Widely Varying Backgrounds

The course becomes particularly problematic when "one course fits all" is the order of the day. Many schools do not have sufficient numbers of students or faculty to offer similar but different versions of the course to accommodate the varying mathematical backgrounds and proficiency levels of their different clientele. Mathematical backgrounds of students within a class may vary from two years of high school algebra to four or more years of college prep mathematics, and their disparate levels of algebraic proficiency is one of the most troublesome aspects of the course.

Most texts begin with a review of a selection of algebraic topics. Before changing our approach at the University of Evansville, sometimes as much as two weeks were devoted to this "preliminary" material. But, for whose benefit? The students with good preparation would have polished up their rusty skills in the process of working on calculus topics to follow, and those students who had not been successful in two years of high school algebra instruction were not likely to suddenly develop those skills in a week or two.

Those deficiencies in algebraic skills continued to plague the entire class. Students continued to be frustrated by getting hung up on calculus problems and/or getting wrong answers, not because they didn't know what they were supposed to do, but because of algebraic difficulties in getting it done. Scores on tests depended too heavily on how many points were deducted for answers that were incorrect because of algebra mistakes. Far too much class time was diverted to questions arising from algebra rather than calculus. The progress of the class toward effective study and understanding of calculus concepts was continually impeded by time and attention devoted to "putting out algebra fires."

III. The Solution: CAS?

We were aware of commonly cited advantages of using computers in the classroom: enliven the course and get students more involved; use computer graphics to support intuition and concepts; allow for more interesting exercises with real-world data; facilitate discovery approaches; provide experience using a computer; and provide a natural writing component in the form of computer laboratory reports. But for us, a major motivation for use of a CAS in the course was an attempt to address the algebra problem.

Heid (1988), working primarily with a calculus class for business students, had gathered evidence that concepts of calculus can be learned without prior or concurrent mastery of algorithms relating to differentiation and integration. It seemed even more likely that understanding of calculus concepts does not depend on algebraic skills per se. It was our thought that use of a CAS might serve to level the "algebraic playing field;" that it would be a time-saver and aid for students with adequate skills, and would be an essential tool for students with inadequate skills. Perhaps it would allowing them to "hurdle" algebra difficulties and get on with the calculus. Our hope and intent was that use of a CAS would allow for more class time directed toward calculus rather than algebra.

IV. Implementation

The CAS we selected was *DERIVE;* it is powerful enough for our needs, modestly price, and user-friendly. We were determined that the software was to be used as a tool for increasing understanding, not to become a focal point of the course. We have found that with some handout material, virtually no class time need be devoted to instruction in the use of *DERIVE*.

Since 1993, calculus classes meet each class day in computer classrooms provided, in part, by an NSF ILS grant. While laboratory assignments requiring structured written reports are a part of most sections of the course, we feel it is even more important that both the instructor and the students see the computer as a common everyday tool. We want them to become comfortable with it, and to acquire some sense of what the computer can and cannot do, what tasks are appropriately given to the computer and what are not.

Frequently, two-part tests are given, one part to be done without the aid of a computer, and another part on which the students may use a computer as they wish. Generally, the part without computers might include definitions, statements of theorems, use of basic formulas, essay questions, etc.; the part with computers might involve interpretation of graphs, more challenging applications and problems involving substantial symbolic manipulation.

Some asked final exam questions that do not involve use of the computer are:

> If $s(t) = -16t^2 + 40t + 10$ gives the height (in feet) of a projectile at time t (in seconds), what is the velocity at $t = 2$ seconds?
>
> Define the derivative of a function at a point.
>
> State the Fundamental Theorem of Integral Calculus
>
> Write a paragraph relating secant lines at a point and their slopes to the tangent line at the point and its slope.
>
> Write a paragraph responding to this question from a person who has studied algebra and geometry but not calculus: “What is calculus"?

On the other hand, some sample final exam questions on which the students may use the computer are:

> Find the point on the graph of $y = ln(x^2 + 4)$ where the slope is greatest, and write an equation of the line tangent to the curve at that point.
>
> Find the absolute minimum and absolute maximum values of $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$ on the interval [-1, 2].

A company determines that the marginal cost per unit produced is given by $C'(x) = 0.003x^2 - 0.2x + 50$. Ignoring fixed costs, **find the total cost** of producing the first 10 units.

Find the area of the region enclosed by the graphs of $y = x^3 - 3x^2$ and $y = 2x^2 - 7x + 3$.

Note that each of these questions requires the student to understand and apply concepts from calculus, while use of a CAS helps them avoid mechanical and algebraic difficulties.

V. Conclusions

We have not gathered hard statistical data involving control groups or any structured experimental design. Hence, conclusions are based only on largely anecdotal evidence, but some general observations are as follows:

Student comments relating to use of computers and *DERIVE* have been generally positive. Most students quickly become accustomed to using the computer; in fact, they grew to rely upon it more than we might like, but not more than we expected.

Students are more likely to approach problems in a greater variety of ways- graphically, numerically, analytically, and combinations of those approaches.

Narrative responses are improved, both in form and in content. This is attributable, we surmise, to student experience gained in writing the laboratory reports, as well as to more time and emphasis given to concept development.

With the effects of algebraic mistakes largely removed, scores on tests more nearly reflect understanding of calculus concepts.

Definitely, more time is being spent on presentation, discussion, and application of calculus concepts, and less time on dealing with algebraic difficulties.

In summary, the CAS approach generally has done what we expected, and is recommended in similar courses where the detrimental effects of poor algebraic skills are to be minimized.